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15.1. INTRODUCTION

Heat is emitted into subsurface ventilation systems from a variety of sources. In the majority of the world’s coal mines, the airstream itself is sufficient to remove the heat that is produced. In deep metal mines, heat is usually the dominant environmental problem and may necessitate the use of large-scale refrigeration plant (Chapter 18). Conversely, in cold climates, the intake air may require artificial heating in order to create conditions that are tolerable for both personnel and equipment.

In Section 9.3.4, quantification of the heat emitted into a mine or section of a mine was required in order to assess the airflow needed to remove that heat. Hence, a sensible place to commence the study of heat flow into mine openings is to classify, analyze and attempt to quantify the various sources of heat. The three major heat sources in mines are the conversion of potential energy to thermal energy as air falls through downcasting shafts or slopes (autocompression), machinery and geothermal heat from the strata. The latter is, by far, the most complex to analyze in a quantitative manner. We shall deal with this separately in Section 15.2, then quantify other sources of heat in Section 15.3.

15.2. STRATA HEAT

15.2.1. Methods of determining strata heat load

In reviewing the literature for means of determining the amount of heat that will be emitted from the strata, the ventilation engineer is faced with a bewildering array of methods varying from the completely empirical, through analytical and numerical, to computer simulation techniques. The basic difficulty is the large number of variables, often interacting with each other, which govern the flow of strata heat into mine airways. These include:

- the length and geometry of the opening
- depth below surface and inclination of the airway
- method of mining
- wetness of the airway surfaces
- roughness of the airway surfaces
- rate of mineral production or rock breaking
- time elapsed since the airway was driven
- volume flow of air
- barometric pressure, and wet and dry bulb temperatures
- virgin (natural) rock temperature
- distance of the workings from downcast shafts or slopes
- geothermic step or geothermic gradient
- thermal properties of the rock
- other sources of heating or cooling such as machines and cooling plant.

With such a variety of parameters, it is hardly surprising that traditional methods of predicting strata heat loads have been empirical. Perhaps the simplest and most common of these has been to quote strata heat flux in terms of heat load per unit rate of mineral production; for example, kW per tonne per day. This can provide a useful and rapid guideline provided that all conditions are similar to those on which the value was based. However, as rate of production is only one of the several variables listed above, it is obvious that this technique can lead to gross errors if it is applied where the value of any one of those variables is significantly different from the original sets of measurements used to establish the kW/tonne/day value.

The more sophisticated empirical techniques extend their range of application by incorporating estimated corrections for depth, distance, age, inlet conditions or, indeed, any of the listed variables considered to be of local importance.
The purely analytical methods of quantifying heat flow from the strata are somewhat limited for
direct practical application because of the complexity of the equations that describe three-
dimensional, time-transient heat conduction. Indeed, they can be downright frightening. However,
the theory that has evolved from analytical research has provided the basis for numerical
modelling which, in turn, has resulted in the development of pragmatic computer simulation
packages for detailed prediction of variations in the mine climate.

A hybrid method has grown out of experience in running climatic simulation packages. It is often
the case that, for particular conditions, some of the input variables have a very limited effect on
the results. By ignoring those weaker parameters it is then sometimes possible to develop simple
equations that give an approximation of the heat flow.

In view of these alternative methodologies, what is the mine environmental engineer to do when
faced with the practical problems of system design? Experience gained from major planning
projects has indicated the following recommended guidelines:

1. If the objective is to plan the further development of an existing mine, or if there are
neighbouring mines working similar deposits at equivalent depths and employing the same
methods of working, then the empirical approach (kW per tonne per day) may be adopted for
the overall strata heat load on the whole mine or major sections of the mine. This
presupposes the existence of data that allow acceptable empirical relationships to be
established and verified.

Employing past and relevant experience in this way provides a valuable and simple means of
arriving at an approximate heat load which, when combined with the methodology of Section
9.3.4, will give an indication whether the heat can be removed by the airflow alone, or if
refrigeration is required.

However, let the user beware. If the proposed mine project deviates in any significant manner
from the conditions in which the empirical data were compiled (check the list of variables at
the beginning of this section), then the results may be misleading. In particular, great caution
should be exercised when employing empirical relationships established in other
geographical regions. A phrase commonly heard at mine ventilation conferences is "what
works there, doesn't work here."

2. The hybrid equations are very useful for rapid approximations of heat flow into specified types
of openings. Dr. Austin Whillier of the Chamber of Mines of South Africa produced many
hybrid equations for easy manual application, including:

(a) Radial heat flow into established tunnels

\[ q = 3.35 \times L \times k^{0.854} \times (VRT - \theta_d) \]  

(15.1)

where

- \( q \) = heat flux from strata (W)
- \( L \) = length of tunnel (m)
- \( k \) = thermal conductivity of rock (W/m°C)
- \( VRT \) = virgin (natural) rock temperature (°C)
- \( \theta_d \) = mean dry bulb temperature (°C)

Throughout this chapter we shall use the symbol \( k \) for thermal conductivity and \( \theta \) for temperature.
(b) **Advancing end of a heading**

\[ q = 6k(L + 4DFA)(VRT - \theta_d) \quad \text{W} \]

where

- \( L \) = length of the advancing end of the heading (m). This should be not greater than the length advanced in the last month. Equation (15.1) may be used for the older sections of the heading.
- \( DFA \) = daily face advance (m).

(c) **One dimensional heat flow towards planar surfaces**

Assuming good convective or evaporative cooling of the surfaces,

\[ q = \frac{A(k \rho C)^{0.5} (VRT - \theta_d)}{t^{0.5}} \quad \text{W} \quad (15.3) \]

where

- \( A \) = area of surface (m²)
- \( \rho \) = rock density (kg/m³)
- \( C \) = specific heat of rock (J/(kg°C))
- \( t \) = time since the surface was exposed (s)

A weakness of these equations is that they each contain the mean dry bulb temperature, \( \theta_d \). As this is initially unknown, it must be estimated. If, when the resulting value of \( q \) is used to determine the temperature rise, the initial estimate of \( \theta_d \) is found to have been significantly in error then the process may be repeated. However, as the hybrid equations promise nothing more than rough approximations there may be little point in progressing beyond a single iteration.

3. For accurate and detailed planning, a mine climate simulation package should be employed. These are computer programs that have been developed to take all of the relevant variables into account. They may be used for single airways or combined into a total underground layout. Climate simulation programs go beyond the calculation of heat load by predicting the effects of that heat on the psychrometric conditions in the mine. The principles of a climate simulation program are outlined in Chapter 16.

For major projects, estimates of heat loads may be based on empirical and hybrid methodologies for initial conceptual layouts, progressing to simulation techniques for detailed planning.

15.2.2. Qualitative observations

Before embarking upon a quantitative analysis of strata heat flow, it will be useful to introduce some of the observable phenomena in a purely qualitative manner.

First, when cool air passes through a level airway, its temperature usually increases. This is caused by natural geothermal heat being conducted through the rock towards the airway, then passing through the boundary layers that exist in the air close to the rock surface. In working areas, the newly exposed rock surfaces are often perceptibly warmer than the air. However, those surfaces cool with time until they may be only a fraction of a degree C higher than the temperature of the air.

If the airway is wet then the increase in dry bulb temperature is less noticeable. Indeed, that temperature may even fall. This is a result of the cooling effect of evaporation. Heat may still emanate from the strata but all, or much of it, is utilized in exciting water molecules to the extent
that they leave the liquid phase and form water vapour. The heat content and wet bulb temperature of the air/vapour mixture then rise because of the internal energy of the added water vapour.

Another observation that can be made in practice is that although the air temperature in main intake arteries rises and falls in sympathy with the surface climate, the temperatures in main returns remain remarkably constant throughout the year. This is because cool air will encourage heat to flow from the rock. However, as the temperature of the air approaches the natural temperature of the rock such heat transfer will diminish. It can, of course, work in reverse. For example, the temperature of the air leaving an intensively mechanized working area may be greater than the local strata temperature. In that case, heat will pass from the air to the rock. The air will cool and, again, approach equilibrium when its temperature equals that of the strata. A mine is an excellent thermostat.

The envelope of rock immediately surrounding a newly driven airway will cool fairly rapidly at first. There will, accordingly, be a relatively high rate of initial heat release into the air. This will decline with time. A well-established return airway may have reached near thermal equilibrium with the surrounding strata. However, the linings and envelope of rock around downcast shafts or main intakes will emit heat during the day when the incoming air is cool and, conversely, absorb heat during the day if the air temperature becomes greater than that of the surrounding envelope of rock. This cyclic phenomenon, sometimes known as the “thermal flywheel” (Stroh, 1979) is superimposed upon longer term cooling of the larger mass of rock around the opening.

The boundary layers that exist within the airflow close to the rock surface act as insulating layers and, hence, tend to inhibit heat transfer between the rock and the main airstream. It follows that any thinning or disturbance of those boundary layers will increase the rate at which heat transfer takes place. This can occur either through a rise in air velocity or because of a greater degree of roughness on the surface (Section 2.3.6).

Finally, although there may be significant rises in the air temperature along intake airways, the most noticeable increase usually occurs in the mineral winning areas. This is because, first, the newly exposed and warm surfaces of both the solid and broken rock give up their heat readily and, secondly, the mechanized equipment that may be concentrated in stopes or working faces.

Having introduced these concepts in a purely subjective manner, we now have a better intuitive understanding from which to progress into a quantified and analytical approach.

15.2.3. Fourier's law of heat conduction

When a steady heat flux, \( q \), passes through a slab of homogeneous material, the temperature will fall from \( \theta_1 \) at entry to \( \theta_2 \) at exit (Figure 15.1). Planes of constant temperature, or isotherms, will exist within the material. Figure 15.1 also shows two isotherms at a short distance, \( dx \), apart and with temperatures \( \theta \) and \( \theta + d\theta \).

The heat flux, \( q \), is proportional to both the orthogonal area, \( A \), through which the heat travels and the temperature difference, \( d\theta \), between isotherms. It is also inversely proportional to the distance, \( dx \), between those isotherms.

Figure 15.1 Linear heat flow.
Hence,

\[ q \text{ is proportional to } - A \frac{d\theta}{dx} \]

where 

- \( q \) = heat flux (W) 
- \( A \) = area through which \( q \) passes (m²), 
- \( \theta \) = temperature (°C) and 
- \( x \) = distance (m)

The negative sign is necessary since \( \theta \) reduces in the direction of heat flow, i.e. \( d\theta \) is negative. To convert this relationship into an equation, a constant of proportionality, \( k \), is introduced, giving

\[
q = -k A \frac{d\theta}{dx} \quad W \quad \text{(Fourier's Law)} \quad (15.4)
\]

\( k \) is termed the thermal conductivity of the material and has units of W/(m°C).

To be precise, \( k \) is a slowly changing function of temperature. It may also vary with the mechanical stress applied to the material. In the strata around mine openings, the effective thermal conductivity of the strata can be significantly different from those given by samples of the rock when measured in a laboratory test (Section 15.2.10). The reasons for such differences include natural or induced fractures in the strata, variations in mineralogy that may be direction dependent, movements of groundwater, radioactive decay and local geothermal anomalies.

15.2.4. Geothermic gradient, geothermal step and thermal conductivity

The crustal plates upon which the continents drift over geological time are relatively thin compared to the diameter of the earth. Furthermore, it is only in the upper skin of those plates that mining takes place at the present time. The geothermal flow of heat emanating from the earth's core and passing through that skin has an average value of 0.05 to 0.06 W/m². It can, of course, be much higher in regions of anomalous geothermal activity.

In Fourier's law, equation (15.4), if we use the value of 0.06 W for each square metre of land surface to give the variation of temperature, \( \theta \), with respect to depth \( D = -x \), then

\[
0.06 = \frac{d\theta}{dD} \quad ^\circ\text{C} \quad m^{-1} \quad (15.5)
\]

The increase of strata temperature with respect to depth is known as the geothermic gradient. In practical utilization, it is often inverted to give integer values and is then referred to as the

\[
\text{Geothermal step} = \frac{dD}{d\theta} \quad m/^\circ\text{C} \quad (15.6)
\]

It is clear from equations (15.5) and (15.6) that as we progress downwards through a succession of strata, the geothermal gradient and geothermal step will vary according to the thermal conductivity of the local material.

Table 15.1 has been assembled from several sources as a guide to the thermal conductivities and corresponding geothermal steps that may be expected for a range of rock types. However, it should be remembered that these parameters are subject to significant local variations. Site specific values should be obtained, preferably from in-situ tests, for any important planning work.
Table 15.1. Typical values of thermal conductivity and geothermal step for a range of rock types.

For air and water in the range 0 to 60°C (Hemp, 1985)

\[
\begin{align*}
    k_a &= 2.2438 \times 10^{-4} T^{0.8353} \\
    k_w &= 0.2083 + 1.335 \times 10^{-3} T
\end{align*}
\]

where \( T \) = Absolute temperature (K)

15.2.5. An analysis of three dimensional heat conduction.

During the practical application of empirical, hybrid or simulation techniques of assessing strata heat loads (Section 15.2.1) the mine environmental engineer need give little conscious thought to the theory of heat conduction or the derivations given in this Section. These analyses are included here because they provide the essential relationships for the development and internal operation of numerical models and, hence, mine climate simulation packages. Readers who are not concerned with the analytical treatment should skip directly to Section 15.3.

Within the envelope of rock that surrounds an underground airway, the temperature varies with both position and time. Our first task is to derive a general relationship that defines the temperature of the rock as a function of location and time. For simplicity, we shall assume that the thermal properties of the rock remain constant with respect to both position and time.

The general equations for heat conduction are readily derived in Cartesian co-ordinates. However, for mine airways, it is more practical to work in cylindrical polar co-ordinates. Figure 15.2 represents a mass of strata surrounding an underground airway. The \( z \) axis represents an airway simply as a line. The position of any point in the strata can be defined by co-ordinates \( z, r \) and \( \varnothing \) where \( r \) is the radial distance from the centreline and \( \varnothing \) is the angle from the horizontal measured in radians.

Consider a small trapezoidal element lying in a thin annulus of rock at a distance \( r \) from the centre line. The length of the element is \( dz \), and its radial height is \( dr \). The inner width is \( r d\varnothing \) increasing to \( (r + dr)d\varnothing \) at its outer limit.
We shall analyse the heat flux passing through the element shown shaded. This can be accomplished by examining, in turn, the partial heat flows in the directions of increasing $r$, $\varnothing$ and $z$, and in that order.
1. The base of the element has an area \( r\, d\phi\, dz \), the curvature being negligible over such short distances. The flux passing through this face is given by Fourier’s Equation (15.4)

\[
dq_{1r} = -k\, r \, d\phi \, dz \frac{\partial \theta}{\partial r} \quad W
\]

The opposite face has an area of \((r + dr)\, d\phi\, dz\) and passes a heat flow of

\[
dq_{2r} = -k\, (r + dr) \, d\phi \, dz \left( \theta + \frac{\partial \theta}{\partial r} \, dr \right) \quad W
\]

Heat retained by the element in the \( r \) direction is

\[
dq_{1r} - dq_{2r} = k\, r \, d\phi \, dz \frac{\partial^2 \theta}{\partial r^2} \, dr + k\, dr \, d\phi \, dz \left( \frac{\partial^2 \theta}{\partial r^2} + \frac{\partial^2 \theta}{\partial r \, \partial r} \, dr \right) = k\, dr \, d\phi \, dz \left( \frac{r \, \partial^2 \theta}{\partial r^2} + \frac{\partial^2 \theta}{\partial r \, \partial r} \, dr \right)
\]

as the term involving \((dr)^2\) is insignificant.

2. The heat flux in the direction of increasing \( \phi \) passes through opposite faces, each of area \( dr\, dz \). The direction is actually \( r\phi \). Hence, Fourier’s equation for heat entering the element is

\[
dq_{1\phi} = -k\, dr \, dz \frac{\partial \theta}{\partial (r\phi)} = -k\, dr \, dz \frac{\partial \theta}{r \, \partial \phi} \quad W
\]

while the heat leaving the element is

\[
dq_{2\phi} = -k\, dr \, dz \frac{\partial}{r \, \partial \phi} \left( \theta + \frac{\partial \theta}{r \, \partial \phi} \, r \, d\phi \right) = -k\, dr \, dz \left( \frac{\partial \theta}{r \, \partial \phi} + \frac{\partial^2 \theta}{r \, \partial \phi^2} \, d\phi \right) \quad W
\]

Heat gain in the \( r\phi \) direction is

\[
dq_{1\phi} - dq_{2\phi} = k\, dr \, dz \, d\phi \frac{\partial^2 \theta}{r \, \partial \phi^2} \quad W \quad (15.8)
\]

3. Repeating the exercise for the \( z \) direction, the opposite faces are trapeziums of height \( dr \) and opposite edges of length \( r\, d\phi \) and \((r + dr)\, d\phi\). the curvature of the lines being insignificant over such small distances. The latter have an average length of

\[
\frac{r\, d\phi + (r + dr)\, d\phi}{2} \quad m
\]

The product of the differentials \( dr\, d\phi \) is negligible compared with \( r\, d\phi \), giving the width to be \( r\, d\phi \) and face area of \( r\, d\phi\, dr \).

The heat entering the element in the \( z \) direction is

\[
dq_{1z} = -k\, r \, d\phi \, dr \frac{\partial \theta}{\partial z} \quad W
\]

and leaving the opposite face,

\[
dq_{2z} = -k\, r \, d\phi \, dr \left( \theta + \frac{\partial \theta}{\partial z} \, dz \right) \quad W
\]
Then the rate of heat accumulated in the $z$ direction is

$$dq_{1z} - dq_{2z} = kr d\phi \, dr \, dz \, \frac{\partial^2 \theta}{\partial z^2} \quad W \quad (15.9)$$

The total rate of heat gain by the element, $dq$, is given by summing equations (15.7, 15.8 and 15.9)

$$dq = kr dr d\phi \, dz \left\{ r \frac{\partial^2 \theta}{\partial r^2} + \frac{\partial \theta}{\partial r} + \frac{1}{r} \frac{\partial^2 \theta}{\partial \phi^2} + r \frac{\partial^2 \theta}{\partial z^2} \right\} \quad W \quad (15.10)$$

The heat gain by the element in time $\partial t$ (where $t =$ time in seconds) can also be expressed as

$$dq = mC \frac{\partial \theta}{\partial t} \quad W$$

where $m =$ mass of element and $C =$ specific heat of the material.

But $m =$ volume x density ($\rho$) = $\rho \, dr \, dz \, r \, d\phi$

Then

$$dq = dr \, dz \, r \, d\phi \, \rho \, C \frac{\partial \theta}{\partial t} \quad W \quad (15.11)$$

Hence, from Equations (15.10 and 15.11)

$$k \left\{ r \frac{\partial^2 \theta}{\partial r^2} + \frac{\partial \theta}{\partial r} + \frac{1}{r} \frac{\partial^2 \theta}{\partial \phi^2} + r \frac{\partial^2 \theta}{\partial z^2} \right\} = \rho \, C \frac{\partial \theta}{\partial t} \quad \frac{W}{m^2} \quad (15.12)$$

or

$$\frac{k}{\rho C} \left\{ \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \theta}{\partial \phi^2} + \frac{\partial^2 \theta}{\partial z^2} \right\} = \frac{\partial \theta}{\partial t} \quad \frac{^\circ C}{s}$$

This fundamental relationship is the general three-dimensional equation for unsteady heat conduction expressed in cylindrical polar co-ordinates.

For most purposes in strata heat conduction towards airways, we can assume that

$$\frac{\partial \theta}{\partial z} = \frac{\partial^2 \theta}{\partial z^2} = 0$$

and that

$$\frac{\partial \theta}{\partial \phi} = \frac{\partial^2 \theta}{\partial \phi^2} = 0$$

These simplifications are based on the premise that the natural geothermic gradient is small compared to the radial variation in temperature around the incremental length of airway, and that the heat conduction is radial along the full length.
Heat flow into subsurface openings

Then

\[
\frac{k}{\rho C} \left( \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} \right) = \frac{\partial \theta}{\partial t} \quad ^\circ \text{C} \quad \text{s}^{-1}
\]

The term \(k/\rho C\) is a constant for the material and is called the thermal diffusivity, \(\alpha\) (\(\text{m}^2/\text{s}\)) giving

\[
\alpha \left( \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} \right) = \frac{\partial \theta}{\partial t} \quad ^\circ \text{C} \quad \text{s}^{-1}
\]

(15.13)

This is the form of the equation normally quoted for radial heat conduction and is the basis on which strata heat flow is determined in mine climate simulation programs. Figure 15.3 gives a graphical illustration of the time transient heat conduction equation.

Figure 15.3  An illustration of the variation of rock temperature with respect to time and distance into the rock.
15.2.6. Solution of the radial heat conduction equation

Having derived equation (15.13) for the time-space variation in temperature around an underground airway, we must now attempt to transform it into a practical procedure to determine strata heat emission into the airway.

In order to facilitate further analysis and generality, it is convenient to express radial distances and time as dimensionless numbers.

Dimensionless radius \( r_d = \frac{r}{r_a} \)

where \( r_a = \) effective radius of the airway (= perimeter / \( 2\pi \))

The actual shape of the cross section has little effect on the influx of strata heat into airways.

Dimensionless time \( F_o = \frac{\alpha t}{r_a^2} \) (this is called the Fourier number)

Equation (15.13) then becomes

\[
\frac{\partial^2 \theta}{\partial (r_d r_a)^2} + \frac{1}{r_d r_a} \frac{\partial \theta}{\partial (r_d r_a)} = \frac{1}{\alpha} \frac{\partial \theta}{\partial (F_o r_a^2 / \alpha)}
\]

As \( r_a \) and \( \alpha \) are constants

\[
\frac{\partial^2 \theta}{r_a^2 \partial (r_d)^2} + \frac{1}{r_d r_a^2} \frac{\partial \theta}{\partial (r_d)} = \frac{1}{\alpha r_a^2 / \alpha} \frac{\partial \theta}{\partial (F_o)}
\]

The \( r_a^2 \) and \( \alpha \) terms cancel, giving the radial heat conduction equation as

\[
\frac{\partial^2 \theta}{\partial (r_d)^2} + \frac{1}{r_d} \frac{\partial \theta}{\partial (r_d)} = \frac{\partial \theta}{\partial (F_o)} \tag{15.14}
\]

The determination of heat flux into the airway commences by applying Fourier’s law (equation (15.4)) to each square metre of rock surface:

\[
q = k \left( \frac{\partial \theta}{\partial r} \right)_s \left( \frac{W}{m^2} \right) \tag{15.15}
\]

where \( \left( \frac{\partial \theta}{\partial r} \right)_s \) is the temperature gradient in the rock but at the rock/air interface (subscript \( s \) for surface).

This same heat flux, \( q \), passes from each square metre of surface through the boundary layers into the main airstream. However, for any given type of surface and flow conditions, the heat transferred through the boundary layers is proportional to the temperature difference across those layers,

i.e. \( q = h (\theta_s - \theta_d) \left( \frac{W}{m^2} \right) \tag{15.16} \)
where $\theta_s =$ temperature of the rock surface (°C) 
$\theta_d =$ dry bulb temperature in the main airstream (°C) and 
$h =$ a heat transfer coefficient ( W/(m$^2$°C) ) that is a function mainly of the air velocity and the characteristics of the rock surface.

We shall discuss the heat transfer coefficient further in Section 15.2.7. However, for the moment let us accept it simply as a constant of proportionality between $q$ and $(\theta_s - \theta_d)$.

Assuming that we know the values of $h$ and $\theta_d$, then we only need the rock surface temperature, $\theta_s$, for equation (15.16) to give us the required heat flux, $q$. The problem turns to one of finding $\theta_s$.

As the heat flux from the strata must be the same as that passing through the rock/air interface, we can combine equations (15.15) and (15.16) to give

$$q = k \left( \frac{\partial \theta}{\partial r} \right)_s = h(\theta_s - \theta_d) \quad \frac{W}{m^2} \quad (15.17)$$

Again, for generality, the temperature gradient at the surface may be expressed in dimensionless form, $G$, where

$$G = \left( \frac{r_a}{(VRT - \theta_d)} \right) \left( \frac{\partial \theta}{\partial r} \right)_s \quad (15.18)$$

where $VRT =$ virgin (natural) rock temperature (°C)

Combining with equation (15.17) gives

$$G(VRT - \theta_d) = \frac{hr_a}{k}(\theta_s - \theta_d) \quad ^\circ C \quad (15.19)$$

The group $h r_a/k$ is known as the Biot Number, $B$, or dimensionless heat transfer coefficient.

Then

$$G(VRT - \theta_d) = B(\theta_s - \theta_d) \quad ^\circ C$$

or

$$\theta_s = \frac{G}{B}(VRT - \theta_d) + \theta_d \quad ^\circ C \quad (15.20)$$

We have now found an expression for $\theta_s$. However, it contains the dimensionless but yet unknown temperature gradient $G$. This may be obtained from a solution of the general radial heat conduction equation (15.13) using Laplace transforms. The mathematics of the solution process can be found in Carslaw and Jaeger (1956). The result involves a series of Bessel functions and is reproduced in Appendix A15.1 at the end of this chapter. Unfortunately, the solution appears even more disconcerting for practical use than the original differential equation (15.13). However, it is now more amenable to numerical integration. The results are shown on Figure 15.4 as a series of curves from which the dimensionless temperature gradient, $G$, can be read for given ranges of Fourier Number, $F_o$, and Biot number, $B$.

An algorithm produced by Gibson (1975) allows $G$ to be determined much more easily than the full numerical integration of the Carslaw and Jaeger solution. Gibson’s algorithm (Appendix A15.2) is suitable for programming into a personal computer and gives an accuracy of within 2 per cent over the majority of the ranges covered in Figure 15.4.
Older solutions to the general equation (15.13) (Carrier, 1940; Goch and Patterson, 1940) assumed that the rock surface temperature was equal to the dry bulb temperature of the air. This ignored the insulating effect of the boundary layers or, put another way, inferred an infinite heat transfer coefficient and, hence, an infinite Biot Number. The uppermost curve on Figure 15.4 represents this bounding condition and gives the same results as the tables produced by Carrier, and Goch and Patterson.

Having established a value of $G$, equations (15.17) and (15.20) then combine to give the required heat flux:

$$ q = \frac{h G}{B} (\text{VRT} - \theta_d) \quad \text{W/m}^2 \quad (15.21) $$

15.2.7. Heat transfer coefficient for airways

In section 15.2.6, we introduced the heat transfer coefficient, $h$, for the rock surface as the 'constant' of proportionality between heat flux across a boundary layer and the corresponding temperature difference between the rock surface and the general airstream:

$$ q = h (\theta_s - \theta_d) \quad \text{W/m}^2 \quad (15.16) $$

Figure 15.4 These curves enable the dimensionless temperature gradient, $G$, in the rock at its surface to be estimated for known Fourier and Biot Numbers.
It is clear from this equation that for a well established airway where the rock surface temperature, $\theta_s$, is very close to the air dry bulb temperature, $\theta_d$, the strata heat flux $q$ will be small. The value of the heat transfer coefficient will then have little effect on climatic variations in the airway. Conversely, for newly exposed surfaces ($\theta_s - \theta_d$) will be relatively high and the heat transfer coefficient will be a significant factor in controlling the flow of heat from the strata.

It can be seen from Figure 15.4 that for Biot Numbers ($B = h_{ra}/k$) of 10 or more, errors in the heat transfer coefficient and, hence, Biot Number will have little effect on the dimensionless temperature gradient, $G$, which in turn controls heat flux into the airway. For typical sizes of modern mine airways, the Biot Number is generally in excess of 5 where its accuracy (and, hence, the accuracy of the heat transfer coefficient) remains of limited consequence. It is, nevertheless, prudent to establish relationships that will allow us to evaluate the heat transfer coefficient.

The heat transfer coefficient is a constant only within the confines of equation (15.16). The overall heat transfer coefficient, $h$, for mine airways is made up of two parts, the convective heat transfer coefficient, $h_c$, and a component of the radiative heat transfer coefficient, $h_r$.

15.2.7.1. Convective heat transfer

In practice the convective heat transfer coefficient changes with those factors that cause variations in the thickness of the boundary layers, i.e. the air velocity and roughness on the surface that produce near-wall turbulence. Hence, the convective heat transfer coefficient depends upon the coefficient of friction, $f$, (or Atkinson friction factor) and the Reynolds Number, $Re$. These parameters were introduced in Section 2.3 as factors that influence airway resistance and losses of mechanical energy.

A rise in near-wall turbulence causes not only increased cross-flow momentum transfer close to the laminar sublayer but also helps transport heat across the turbulent boundary layer into the mainstream. There is a close analogy between heat and momentum transfer across boundary layers. This has engaged the attention of researchers since the time of Osborne Reynolds and many equations relating heat transfer coefficients to fluid properties and flow regimes have been proposed for both hydraulically smooth and rough surfaces.

A relationship that has shown itself to agree with practical observations both underground (Mousset-Jones et al, 1987; Danko et al, 1988) and in scale models (Deen, 1988) is based on earlier work by Nunner (1956). For application in mine ventilation, it may be stated as

$$N_u = \frac{0.35 f \frac{Re}{1 + 1.592 \{15.217 f Re^{0.2} - 1\}/Re^{0.125}}}{\text{dimensionless}}$$

(15.22)

The convective heat transfer coefficient, $h_c$, is then given as

$$h_c = \frac{Nu \times k_a}{d} \text{ W/(m°C)}$$

(15.23)

where

$$k_a = \text{thermal conductivity of air } (2.2348 \times 10^{-4} T^{0.8353} \text{ W/(m°C) } , T \text{ being the absolute temperature in degrees Kelvin})$$

and

$$d = \text{hydraulic mean diameter (m)}$$

The Nusselt Number ($N_u = h_c d/k_a$) is simply a means of expressing the heat transfer coefficient as a dimensionless number in order to extend the generality of equation (15.22). It is analogous to the Biot Number but is referred to the air and the characteristics of the airway rather than the rock.
Here again, equations (15.22) and (15.23) may be programmed into a calculator or personal computer for a rapid determination of the convective heat transfer coefficient. For manual estimation, the Nusselt Number may be read directly from Figure 15.5 for any given values of $f$ and $Re$. The background to equation (15.22) is given in Appendix A15.3.

A further phenomenon that has been observed experimentally and subjected to theoretical analyses is a tendency for the convective heat transfer coefficient to decrease as the wall temperature rises for any given value of Reynolds Number. This is caused by a thickening of the temperature or thermal boundary layer and, hence, giving an enhanced insulating effect against heat transfer between the wall and the main airstream. However, the actual heat transfer will normally increase because of the greater value of $(\theta_s - \theta_d)$ in equation (15.16).

15.2.7.2. Radiative heat transfer

In addition to convective effects, heat may be lost from a rock surface by radiation. Oscillations in atomic energy levels at the surface produce electromagnetic radiation. This propagates through space until it reaches another solid surface where some of it is reflected, the remainder returning.
to thermal energy by excitation of atoms on the receiving surface. Additionally, some gases, including water vapour and carbon dioxide, will absorb a fraction of the thermal radiation causing a rise in temperature of the gas. Elemental gases such as oxygen and nitrogen, the major constituents of air, are not affected in this way.

According to the Stefan-Boltzmann equation, the net heat transfer by radiation from a hotter to a cooler surface is given as

$$ q_r = 5.67 \times 10^{-8} (T_1^4 - T_2^4) A F_{ev} \text{ W} \quad (15.24) $$

where \( T_1 \) and \( T_2 \) are the absolute temperatures of the hotter and cooler surfaces respectively (K), \( A \) = the smaller of the two surfaces (m), and \( 5.67 \times 10^{-8} \) = the Stefan-Boltzmann constant. (Wm\(^{-2}\)K\(^{-4}\)) (15.25)

The parameter \( F_{ev} \), combines the thermal emissivity of the surfaces and the view factor which quantifies the degree to which the surfaces can “see” each other (Whillier, 1982).

In subsurface environmental engineering, a more practical relationship for radiant heat transfer (in Watts per square meter) from airway surfaces is

$$ q_r = h_r (\theta_a - \theta_d) F_{ev} \text{ W/m}^2 \quad (15.26) $$

where \( h_r \) = radiative heat transfer coefficient [W/(m\(^2\)C)]

The emissivity of polished metal surfaces is fairly low. However, for rough natural surfaces it is usually more than 0.95. Furthermore, the view factor of each unit area of surface to water vapour in the passing airstream, or to much of the nearby rock surface, also approaches unity. Hence, we may approximate \( F_{ev} \) to 1.

The radiative heat transfer coefficient varies from 5 to 7 W/(m\(^2\)C) in the range of surface temperatures 10 to 40 °C. More precisely, it may be estimated as

$$ h_r = 4 \times 5.67 \times 10^{-8} \times T_{av}^3 \text{ W/(m}^2\text{C)} \quad (15.27) $$

where \( T_{av} \) = Average absolute temperature of the two surfaces (K). In practice, this may be taken as the dry bulb temperature of the air.

For a dry airway, the temperature of the rock surface will remain the same around any given perimeter. Hence, \( T_1 = T_2 \) in equation (15.24) and there will be no net transfer of radiant heat between rock surfaces. If part of the surface is wet, however, radiant heat will pass from the dry areas to the cooler wet surface. This will cause a slight increase in strata heat transfer from dry areas. However, the small rise in temperature of the wet surfaces will result in a slightly diminished flow of strata heat to those surfaces. Although the two may not balance, the net effect is small. Radiative heat transfer between rock surfaces is usually ignored.

The amount of thermal radiation absorbed by water vapour in the air varies exponentially with the product of the vapour content and the distance travelled by the radiation through the air/vapour mixture. At a vapour content of 0.019 kg/kg dry air, 22 per cent of the radiant heat will be absorbed within 3 metres and 47 per cent within 30 metres. The remaining radiation that is not absorbed will be received on other rock surfaces and, again, will have little impact on the mine climate.

The radiative heat transfer coefficient, \( h_r \), is usually considerably lower than the convective heat transfer coefficient, \( h_c \), in mine airways and may, indeed, be of the same order as the uncertainty
An estimate of the fraction of thermal radiation absorbed by water vapour in the air may be obtained from

$$a_b = 0.104 \ln(147 \times L)$$  \hspace{1cm} (15.28)

where $\ln$ = natural logarithm

$X$ = water vapour content of air (kg/kg dry air) (Section 14.2.2.)

and $L$ = distance travelled by the radiation through the air (path length, m)

This equation is based on curve fitting to empirical data.

The mean path of the electromagnetic waves propagating from a given point on a rock surface and before they strike another rock surface will depend upon the geometry of the mine opening. For a mine airway a typical mean path length may be of the order of three times the hydraulic mean diameter.

The effective radiant heat transfer coefficient for thermal radiation between the rock and the air becomes $a_b h_r$.

The overall heat transfer coefficient is then

$$h = h_c + a_b h_r \hspace{1cm} W/(m^2 \cdot °C)$$

Analyses can also be carried out for the absorption of thermal radiation by carbon dioxide and dust particles. However, under normal conditions the corresponding rates of thermal absorption are very small.

**Example.**
The dry bulb temperature of the air in a 4m by 3m underground opening is 26 °C. The corresponding moisture content is 0.015 kg/kg dry air. Estimate the effective radiant heat transfer coefficient.

**Solution.**

Hydraulic mean diameter,

$$d_h = \frac{4 \times \text{Area}}{\text{perimeter}} = \frac{4 \times 12}{14} = 3.43 \hspace{0.5cm} m$$

Assume the mean path length of the radiation to be

$$3 d_h = 3 \times 3.43 = 10.3 \hspace{0.5cm} m$$

Fraction of radiation absorbed (from equation (15.28))

$$a_b = 0.104 \ln(147 \times 0.015 \times 10.3) = 0.32$$

From equation (15.27)

$$h_r = 4 \times 5.67 \times 10^{-8} \times (273.15 + 26)^3 = 6.1 \hspace{1cm} W/(m^2 \cdot °C)$$

Then the effective radiant heat transfer coefficient is

$$a_b h_r = 0.32 \times 6.1 = 1.95 \hspace{1cm} W/(m^2 \cdot °C)$$
15.2.8. Summary of procedure for calculating heat flux at dry surfaces

Sections 15.2.5 to 15.2.7 have detailed the derivation of relationships that describe the radial flow of strata heat through the rock towards a mine opening and across a dry surface into the main airstream. As often occurs in engineering, the application of those relationships is straightforward compared with the theoretical analyses that have produced them.

Before moving on to consider heat exchange at wet surfaces, it is convenient to summarize the procedure for calculating the emission of strata heat across a dry surface and to illustrate that procedure by a case study.

Calculation procedure for dry surface

1. **Assemble the data:**
   - airway dimensions (m) .
   - coefficient of friction, \( f \) (= Atkinson friction factor/0.6)
   - age of airway, \( t \) (seconds) .
   - airflow \( Q \) (m\(^3\)/s) .
   - mean dry bulb temperature of air, \( \theta_d \) (°C)
   - wet bulb temperature, \( \theta_w \) (°C )
   - barometric pressure, \( P \) (Pa) .
   - rock thermal properties
     - thermal conductivity, \( k_r \) (W/m°C)
     - density, \( \rho_r \) (kg/m\(^3\))
     - specific heat, \( C_r \) (J/kg °C)
     - diffusivity, \( \alpha_r = \frac{k_r}{\rho_r C_r} \) (m\(^2\)/s)
     - virgin rock temperature, VRT (°C)

2. **Determine derived parameters:**
   - cross sectional area, \( A \) (m\(^2\))
   - perimeter of airway, \( per \) (m)
   - hydraulic mean diameter, \( d_h = \frac{4A}{per} \) (m)
   - effective radius, \( r_a = \frac{per}{(2\pi)} \) (m)
   - Reynolds' Number, \( Re \). For the purposes of this procedure, \( Re \) may be calculated from the approximation \( Re = \frac{268,000 Q}{per} \)
   - moisture content of air, \( X \) (kg/kg dry air) (from Section 14.6)
   - mean radiation path length, \( L \) (m)

3. **Determine the Nusselt Number, \( N_u \)** either from Figure 15.5 or from equation (15.22): i.e.

   \[
   N_u = \frac{0.35 f Re}{1 + 1.592 (15.217 f Re^{0.2} - 1)/Re^{0.125}}
   \]

4. **Determine the overall heat transfer coefficient, \( h \):**
   (a) Convective heat transfer coefficient, \( h_c \)

   \[
   h_c = 0.026 \frac{N_u}{d_h} \text{ W/(m}^2\text{°C)}
   \]

   where 0.026 W/(m\(^2\)°C) = thermal conductivity of air
(b) Effective radiative heat transfer coefficient, \(a_b h_r\)

\[ h_r = 22.68 \times 10^{-8} (273.15 + \theta_d)^3 \text{ W/(m}^2\text{°C)} \]

Absorption fraction \(a_b = 0.104 \ln(147 X L)\)

Effective radiative heat transfer coefficient = \(a_b h_r\)

(c) Overall heat transfer coefficient, \(h = h_c + a_b h_r\) (W/m\(^2\)°C)

5 Calculate Biot Number, \(B\):

\[ B = \frac{h r_a}{k_r} \] (dimensionless)

6. Calculate Fourier Number, \(F_o\):

\[ F_o = \frac{\alpha_r t}{r_a^2} \] (dimensionless)

7. Determine dimensionless temperature gradient in the rock but at the surface, \(G\), either from Figure 15.4 or from Gibson’s algorithm (Appendix A15.2).

8 Determine heat flux, \(q\):

\[ q = h \frac{G}{B} (\text{VRT} - \theta_d) \] (W/m\(^2\))

9. Calculate heat emission into airway:

\[ \frac{q \times \text{per} \times \text{length of airway}}{1000} \] (kW)

Case Study
This case study illustrates not only the calculation procedure but also typical magnitudes of the variables. The purpose of the exercise is to determine the strata heat that will flow into an incremental length of dry airway. The stages of calculation are numbered to coincide with the steps of the procedure given above.

1. Given data:
   - airway dimensions
     \(\text{width} = 3.5 \text{ m, height} = 2.5 \text{ m, length} = 20 \text{ m}\)
   - Atkinson friction factor (at \(\rho_a = 1.2 \text{ kg/m}^3\)) = 0.014 \text{ kg/m}^3
     i.e. coefficient of friction, \(f = 0.014/0.6 = 0.0233\) (dimensionless).
   - airway age = 3 months
     \[ t = \frac{365}{4} \times 24 \times 3600 = 7.884 \times 10^6 \text{ seconds} \]
   - airflow, \(Q = 30 \text{ m}^3/\text{s}\).
   - dry bulb temperature in airway, \(\theta_d = 25 \text{ °C}\)
   - from psychrometric data, the moisture content of the air has been determined to be \(X = 0.01 \text{ kg/kg dry air}\).
• rock thermal properties:
  conductivity, \( k_r = 4.5 \text{ (W/m°C)} \)
density, \( \rho_r = 2200 \text{ kg/m}^3 \)
specific heat, \( C_r = 950 \text{ J/(kg°C)} \)
diffusivity, \( \alpha_r = \frac{4.5}{2200 \times 950} = 2.153 \times 10^{-6} \text{ m}^2/\text{s} \)
  virgin rock temperature, \( \text{VRT} = 42 ^\circ \text{C} \)

2. Further derived parameters:
• cross-sectional area, \( A = 3.5 \times 2.5 = 8.75 \text{ m}^2 \)
• perimeter, \( \text{per} = 2(3.5 + 2.5) = 12 \text{ m} \)
• hydraulic mean diameter, \( d_h = \frac{4A}{\text{per}} = \frac{4 \times 8.75}{12} = 2.917 \text{ m} \)
• effective radius, \( r_a = \frac{\text{per}}{2\pi} = \frac{12}{2\pi} = 1.910 \text{ m} \)
• Reynolds' Number, \( \text{Re} = 268000 \times \frac{30}{12} = 670000 \)
• mean radiation path length, \( L \), is taken as \( 3d_r = 3 \times 2.917 = 8.751 \text{ m} \)

3. The Nusselt Number, \( N_u \), at \( \text{Re} = 670000 \) and \( f = 0.0223 \) is estimated from Figure 13.5 to be 2400. Alternatively, it may be calculated as
\[
N_u = \frac{0.35 \times 0.0233 \times 670000}{1 + 1.592(15.217 \times 0.0233 \times 670000^{0.2} - 1)/670000^{0.125}} = 2433 \text{ (dimensionless)}
\]

4. Overall heat transfer coefficient, \( h \)
   (a) convective heat transfer coefficient, \( h_c \)
\[
h_c = \frac{0.026 N_u}{d_n} = \frac{0.026 \times 2433}{2.917} = 21.69 \text{ W/(m}^2 \text{°C)}
\]
   (b) effective radiative heat transfer coefficient, \( a_b h_r \)
\[
h_r = 22.68 \times 10^{-8} (273.15 + 25)^3 = 6.01 \text{ W/(m}^2 \text{°C)}
\]
  absorption fraction
\[
a_b = 0.104 \ln(147 \times 0.01 \times 8.751) = 0.266
\]
\[
a_b h_r = 0.266 \times 6.01 = 1.60
\]
   (c) overall heat transfer coefficient,
\[
h = h_c + a_b h_r = 21.69 + 1.60 = 23.29 \text{ W/(m}^2 \text{°C)}
\]
This result illustrates the limited effect of thermal radiation.

5. Biot Number, \( B \)
\[
B = \frac{h r_a}{k_r} = \frac{23.29 \times 1.91}{4.5} = 9.885 \text{ (dimensionless)}
\]

6. Fourier Number, \( F_o \)
\[
F_o = \frac{\alpha_r t}{r_a^2} = \frac{2.153 \times 10^{-6} \times 7.884 \times 10^6}{(1.91)^2} = 4.653 \text{ (dimensionless)}
\]
7. Dimensionless rock temperature at the surface, \( G \), for \( B = 9.885 \) and \( F_o = 4.653 \). This may be read from Figure 15.4 or computed from Gibson’s algorithm (Appendix A15.2) as 0.60. It is clear from the graphical method that non-excessive errors in \( B \) (and, hence, heat transfer coefficient) will have a very limited effect on \( G \) for the conditions cited in this case study.

8. Heat flux

\[
q = h \frac{G}{B} (\text{VRT} - \theta_d) = 23.29 \times \frac{0.60}{9.885} (42 - 25) = 24 \quad \text{W/m}^2
\]

9. Heat emission into the 20m length of airway

\[ q = q \times \text{rock surface area} \quad \text{W} \]

or

\[ \frac{24.0 \times 12 \times 20}{1000} = 5.76 \quad \text{kW} \]

15.2.9. Heat transfer at wet surfaces

The majority of underground openings have some degree of water evaporation or condensation occurring on rock surfaces even if no liquid water is visible. The cooling effect of evaporation or the heating effect of condensation on the rock surface will then result in a reduction or increase, respectively, of the wall surface temperature. This, in turn, will modify the strata heat flow toward that surface.

Figure 15.6 shows the heat balance that must exist for thermal equilibrium at the rock/air interface.

\[
q = q_L + q_{\text{sen}} \quad \text{W/m}^2 \quad (15.29)
\]

where

- \( q \): strata heat flux (W/m\(^2\))
- \( q_L \): latent heat transfer (W/m\(^2\))
- \( q_{\text{sen}} \): sensible heat transfer (W/m\(^2\))

In practice, any of these heat flows may be positive or negative. Indeed, a common situation occurs when the wet surface temperature, \( \theta_{ws} \), is less than the dry bulb temperature of the air, \( \theta_d \). In this case, sensible heat will pass from the air to water on the rock surface; \( q_L \) must then accommodate the combined values of \( q \) and \( q_{\text{sen}} \).

In order to prevent unnecessary repetition, we shall assume evaporation in the following analysis. The same logic applies for condensation except that \( q_L \) is negative.
Much of the theory pertaining to heat transfer at dry surfaces applies, also, to wet surfaces. In particular, let us re-examine equation (15.21) which was derived for a dry surface.

\[ q = h_c \frac{G}{B} (VRT - \theta_d) \quad \text{ref (15.21)} \]

(We shall ignore the small effect of radiative heat transfer. Hence, the overall heat transfer coefficient, \( h \), becomes the convective heat transfer coefficient, \( h_c \)).

Consider the functional dependence of each parameter in this equation in turn:

\[ h_c \] depends upon \( f, \text{Re}, k_a \) and \( d \) (equations (15.22 and 15.23))
\[ B \] depends upon \( h_c, r_a \) and \( k_r \) (equation (15.19))
\[ G \] depends upon the Fourier Number, \( F_o = \frac{\alpha_r t}{r_a^2} \) and the Biot Number, \( B \)

and VRT, the virgin rock temperature, is constant for any fixed location.

(Subscript \( r \) refers to the rock and \( a \) to the air or airway).

Notice that none of these parameters depends upon time transient temperatures within the rock nor whether the surface is wet or dry. They have exactly the same values for both wet and dry surface conditions. It is particularly significant that the dimensionless temperature gradient in the rock at the surface, \( G \), is independent of the surface temperature - another illustration of the value of dimensionless numbers. Hence, the relationships derived earlier for \( h_c, B \) and \( G \) for a dry surface apply equally well for a wet surface. However, the evaporative cooling at a wet surface has not been taken into account and this must cause the strata heat flux, \( q \), to increase.

In order to compensate for evaporation while continuing to use equation (15.21) we could select a value of \( \theta_d \) reduced sufficiently such that the value of \( q \) given by equation (15.21) becomes equal to the heat flux actually passing through the wet surface. Let us name that reduced temperature the effective dry bulb temperature, \( \theta_{ef} \).

Then for the wet surface:

\[ q = h_c \frac{G}{B} (VRT - \theta_{ef}) \quad \text{W/m}^2 \quad \text{ref (15.30)} \]

Provided that we replace \( \theta_d \) by \( \theta_{ef} \) then the equations for heat transfer at a dry surface become applicable also for a wet surface. The problem now is to find the value of \( \theta_{ef} \).

In order to evaluate \( q_c \) and \( q_{sen} \), we shall require the wet surface temperature, \( \theta_{ws} \). The form of equation (15.20) gives a relationship between \( \theta_{ws} \) and \( \theta_{ef} \).

\[ \theta_{ws} = \frac{G}{B} (VRT - \theta_{ef}) + \theta_{ef} \quad ^\circ C \quad \text{(15.31)} \]

This transposes to

\[ \theta_{ef} = \left\{ \theta_{ws} - \frac{G}{B} VRT \right\} \frac{B}{B - G} \quad ^\circ C \quad \text{(15.32)} \]
Substituting for $\theta_{ef}$ in equation (15.30) and simplifying gives

$$q = h_c \frac{G}{(B - G)} (VRT - \theta_{ws}) \frac{W}{m^2} \quad (15.33)$$

We now have an equation for the strata heat flux where everything is known except the wet surface temperature, $\theta_{ws}$. Let us establish similar equations for $q_L$ and $q_{sen}$, the other two terms in the heat balance of equation (15.29).

The **sensible heat** is given simply as

$$q_{sen} = h_c (\theta_{ws} - \theta_d) \quad W/m^2 \quad \text{(see equation (15.16))} \quad (15.34)$$

The **latent heat** term, $q_L$, is given by

$$q_L = 0.0007 h_c L_{ws} \frac{(e_{ws} - e)}{P} \quad W/m^2 \quad (15.35)$$

where $L_{ws}$ = latent heat of evaporation at the wet surface temperature (J/kg)

$e_{ws}$ = saturated vapour pressure at the wet surface temperature (Pa)

$e$ = actual vapour pressure in the mainstream (Pa) and

$P$ = barometric pressure (Pa)

The derivation of equation (15.35) is given in Appendix A15.4. Values of $L_{ws}$, $e_{ws}$, and $e$ may be determined from the relationships given in Section 14.6. In particular $L_{ws}$ and $e_{ws}$ are functions of $\theta_{ws}$ only while $e$ depends upon the wet and dry bulb temperatures of the mainstream, $\theta_w$ and $\theta_d$, and the barometric pressure, $P$.

Equations (15.33, 15.34, and 15.35) allow $q$, $q_{sen}$ and $q_L$ to be determined on the basis of known mainstream conditions and an assumed wet surface temperature, $\theta_{ws}$. If the assumed value of $\theta_{ws}$ were correct, then the heat balance

$$q = q_{sen} + q_L \quad \text{(from equation (15.29)}$$

would hold. However, an error in $\theta_{ws}$ will cause a corresponding error $E$ in the heat balance:

$$E = q - (q_{sen} + q_L) \quad (15.36)$$

The wet surface temperature, $\theta_{ws}$, may now be corrected according to the value of $E$ and the process repeated iteratively until $E$ becomes negligible. All heat flows are then defined.

**Case Study**

In order to illustrate the effect of water on heat flow across a rock surface, we shall use the same data given for the case study in Section 15.2.8 but now assuming that the surface is fully wetted.

To recap on the data:

1. **Constants**:
   - Fourier Number, $F_o = 4.653$ (dimensionless)
   - Convective heat transfer coefficient, $h_c = 21.69 \, (W/m^2°C)$
   - Biot Number, $B = h_c \frac{f_a}{k_r} = 21.69 \times \frac{1.91}{4.5} = 9.207$ (dimensionless)
The value of $B$ has changed by some 7 percent as we are now ignoring radiative effects i.e. using $h_c$ instead of the overall heat transfer coefficient, $h$. However, when we employ $F_o$ and the modified $B$ on Figure 15.4 or in Gibson's algorithm (Appendix A15.2), the value of dimensionless temperature gradient $G$ is not significantly different from the 0.6 obtained previously.

2 Psychrometric Data
The psychrometric relationships used here are listed in Section 14.6. In addition to the dry bulb temperature of 25°C, we shall also require the wet bulb temperature, $\theta_w$, and the barometric pressure, $P$, of the mainstream. For this case study we shall take these to be 17.9°C and 100 kPa respectively. Additionally, we need to assume a value for the temperature of the wet surface. As a first estimate, let us assume $\theta_{ws}$ to be the mean of the wet and dry bulb temperatures in the airstream, i.e. $\theta_{ws} = (25.0 + 17.9)/2 = 21.45$ °C.

(a) For the main airstream:

Saturated vapour pressure, $e_{\text{sat,air}} = 610.6 \exp \left( \frac{17.27 \times 17.9}{237.3 + 17.9} \right) = 2050.4$ Pa

Saturated moisture content, $X_{\text{sat,air}} = \frac{0.622 \times 2050.4}{100000 - 2050.4} = 0.01302$ kg/kg dry air

Latent heat of evaporation at wet bulb temperature, $L_w,\text{air} = (2502.5 - 2.386 \times 17.9) \times 10^3 = 2459.79 \times 10^3$ J/kg

Sigma heat, $S_{\text{air}} = \left( 2459.79 \times 10^3 \times 0.01302 \right) + \left( 1005 \times 17.9 \right) = 50016$ J/kg

Actual moisture content $X_{\text{air}} = \frac{50016 - \left( 1005 \times 25 \right)}{2459.79 \times 10^3 + 1884(25 - 17.9)} = 0.01006$ kg/kg dry air

Actual vapour pressure, $e = \frac{100000 \times 0.01006}{0.622 + 0.01006} = 1592$ Pa

(b) For the wet surface (at temperature $\theta_{ws}$)

Saturated vapour pressure, $e_{ws} = 610.6 \exp \left( \frac{17.27 \times 21.45}{237.3 + 21.45} \right) = 2556$ Pa

Latent heat of evaporation, $L_{ws} = (2502.5 - 2.386 \times 21.45) \times 10^3 = 2451.3 \times 10^3$ J/kg

3 Heat flows:

Strata heat, $q = \frac{h_c G}{B - G} (VRT - \theta_{ws}) = \frac{21.69 \times 0.6}{(9.207 - 0.6)} (42 - 21.45) = 31.1$ W/m²

Sensible heat, $q_{\text{sens}} = h_c (\theta_{ws} - \theta_d) = 21.69(21.45 - 25) = -77.0$ W/m²
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Latent heat,

\[ q_L = 0.0007 n_L p \frac{L_{ws}}{P} \ (e_{ws} - e) \]

\[ = 0.0007 \times 21.69 \times 2451.3 \times 10^{-3} \times \frac{(2556 - 1592)}{100000} = 358.8 \ W/m^2 \]

4 Correction of \( \theta_{ws} \):
The error in the heat balance is

\[ E = q - (q_{sen} + q_L) = 31.1 - (-77.0 + 358.8) = -251 \ W/m^2 \]

Corrected value of \( \theta_{ws} = 21.45 + (-251) \times 0.01 = 18.94 ^\circ C \)

The correction function used here is 0.01 E. The constant 0.01 is chosen to give a reasonable rate of convergence. While mathematical techniques are available to find optimum correction functions for given iterative procedures, a more pragmatic approach (assuming computer availability) is simply to experiment with a set of typical data until satisfactory convergence is achieved.

The procedure from steps 2b to 4 is repeated iteratively until the absolute value of E becomes less than 0.01 W/m². This is clearly a task for a personal computer or programmable calculator. Programming the equations and logic into an algorithm is a useful student exercise. The results of running such an algorithm on this particular example are shown on Table 15.2. Balance was achieved to within 0.01 W/m² after nine iterations. The first line of the table gives values that are in sensible agreement with the hand calculations given above.

<table>
<thead>
<tr>
<th>Iteration</th>
<th>Wet surface temperature ( \theta_{ws} ) °C</th>
<th>Strata heat ( q ) W/m²</th>
<th>Latent heat ( q_L ) W/m²</th>
<th>Sensible heat ( q_{sen} ) W/m²</th>
<th>Error ( E ) W/m²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>21.45</td>
<td>31.1</td>
<td>358.6</td>
<td>-77.0</td>
<td>-250.49</td>
</tr>
<tr>
<td>2</td>
<td>18.95</td>
<td>34.9</td>
<td>222.7</td>
<td>-131.4</td>
<td>-56.43</td>
</tr>
<tr>
<td>3</td>
<td>18.38</td>
<td>35.8</td>
<td>194.5</td>
<td>-143.6</td>
<td>-15.11</td>
</tr>
<tr>
<td>4</td>
<td>18.23</td>
<td>36.0</td>
<td>187.1</td>
<td>-146.9</td>
<td>-4.19</td>
</tr>
<tr>
<td>5</td>
<td>18.19</td>
<td>36.1</td>
<td>185.0</td>
<td>-147.8</td>
<td>-1.17</td>
</tr>
<tr>
<td>6</td>
<td>18.18</td>
<td>36.1</td>
<td>184.4</td>
<td>-148.0</td>
<td>-0.33</td>
</tr>
<tr>
<td>7</td>
<td>18.17</td>
<td>36.1</td>
<td>184.3</td>
<td>-148.1</td>
<td>-0.09</td>
</tr>
<tr>
<td>8</td>
<td>18.17</td>
<td>36.1</td>
<td>184.2</td>
<td>-148.1</td>
<td>-0.03</td>
</tr>
<tr>
<td>9</td>
<td>18.17</td>
<td>36.1</td>
<td>184.2</td>
<td>-148.1</td>
<td>-0.01</td>
</tr>
</tbody>
</table>

Table 15.2. Successive values computed during the iterative process for a wet surface.

At balance, 36.1 W of strata heat arrive at each square metre of wet surface. Simultaneously, 148.1 W of sensible heat pass from the air to the wet surface \( (q_{sen} \) is negative). Hence, the sum of the two, 184.2 W/m² is emitted as latent heat to the air. The temperature of the wet surface is 18.17 °C.

The effect of water on the surface has now been quantified. The net heat emission of 36.1 W/m² compares with 24.0 W/m² for the corresponding (dry) surface of the earlier case study. For the 20 m
length of airway at a perimeter of 12 m, this translates into a heat load of \( \frac{36.1 \times 20 \times 12}{1000} = 8.66 \) kW compared with 5.76 kW when the surface was dry. However, whereas the 5.76 kW of heat from the formerly dry surface was sensible heat, the net 8.66 kW emitted from the wet surface now takes the form of latent heat. This results in significant increases in the sigma heat, wet bulb temperature and moisture content of the air.

This Case Study has illustrated that the determination of heat flux from a wet airway surface involves lengthy calculations including an iterative procedure. The integrity of those calculations can be checked through the effective dry bulb temperature, \( \theta_{ef} \), which was introduced as a device to assist in the analysis. Equation (15.32) gave:

\[
\theta_{ef} = \left\{ \theta_{ws} - \frac{G}{B} \frac{\text{VRT}}{B - G} \right\} \frac{B}{B - G}
\]

Using the value of wet surface temperature \( \theta_{ws} = 18.17^\circ\text{C} \), determined through the iterative procedure this becomes

\[
\theta_{ef} = \left\{ 18.17 - \frac{0.6}{9.207} \right\} \frac{9.207}{9.207 - 0.6} = 16.51^\circ\text{C}
\]

Recall that \( \theta_{ef} \) is what the value of the dry bulb temperature in a completely dry airway would have to be in order to give the same value of heat flux as the actual wet airway. Equation (15.30) or (15.21) for this hypothetical dry airway give a heat flux of

\[
q = h_c \frac{G}{B} (\text{VRT} - \theta_{ef}) = 21.69 \times \frac{0.6}{9.207} (42 - 16.51) = 36 \frac{\text{W}}{\text{m}^2}
\]

and is in good agreement with the 36.1 W/m² obtained for the actual wet airway.

It is clear from these case studies that the analytical determination of strata heat flux into subsurface openings does not lend itself to manual calculation. Chapter 16 discusses simulation programs which can readily be employed for rapid and detailed predictions of the underground climate. However, the engineer who uses such programs is utilizing the theory and procedures that have been described in this Section.

15.2.10. In-situ measurement of rock thermal conductivity

The experimental determination of the thermal conductivity of solids depends essentially upon Fourier’s law (equation (15.4)). A source of heat is applied to one side of a sample. The heat flux across a known area and the corresponding temperature difference across the sample are measured leaving the thermal conductivity as the only unknown.

A number of laboratory test rigs have been developed for the determination of the thermal conductivities of rock samples. However, for subsurface environmental engineering, it is preferable to measure rock thermal conductivity in-situ. The reason for this is the significant difference that may be found between an in-situ value and the mean of numerous samples measured in the laboratory. If the rock is homogeneous and unfractured around mine openings, and if workings are above the water table then good correlation may be achieved between laboratory tests and in-situ determinations. However, in the more usual situation of mining induced fractures in addition to the existence of natural fracture patterns, and particularly in the presence of groundwater migration, the effective thermal conductivity may be more than double that indicated by small and unfractured laboratory samples (Mousset-Jones, 1984).

Water has a thermal conductivity of 0.62 W/(m°C) which is considerably lower than that of most rocks. Hence, the presence of still connate water in fissured or porous media will tend to inhibit
heat flow. Unfortunately, water in the strata surrounding subsurface openings is seldom stationary and, dependent upon the rate of water migration, the transmission of heat by moving water may be far greater than that resulting from conduction through the rock (see, also, section 15.3.3.). This can result in considerable increases in the effective thermal conductivity of the strata.

The simplest method of determining rock thermal conductivity in-situ is to use natural geothermal energy as the heat source. If possible, an airway should be found that is fairly well established, but sufficiently long to give a readily measured rise in wet bulb temperature of the air over its length. A thermocouple circuit to indicate the rise in wet bulb temperature over the selected length of airway will give better accuracy than mercury in glass thermometers. From a cross section within the chosen length, four boreholes should be drilled radially outward to a depth of at least 10 m. These should each be fitted with strings of thermocouples or other type of temperature transducers, positioned to indicate the variation in temperature along the hole. Precautions should be taken to ensure good thermal contact between the transducers and the side of the borehole, and to prevent convection currents of air or water within the hole.

Returning to Figure 15.2, consider an elongated annulus of rock at radius $r$ and length $Y$. The heat flux, $q$, passes radially through the annulus which has an orthogonal area $2\pi r Y$ and thickness $dr$. Fourier's law (equation (15.4)) gives

$$q = -k_r \frac{d\theta}{dr}$$

($dr$ is negative as $r$ is measured outward from the airway while $q$ is usually directed toward the airway).

Then

$$\frac{dr}{r} = \frac{2\pi k_r Y}{q} \frac{d\theta}{\theta}$$

If we assume steady state conditions, $q$ is constant for all values of $r$. We can then integrate to give

$$\ln(r) = \frac{2\pi k_r Y}{q} \theta + \text{constant} \quad (15.37)$$

From the measurements taken in a borehole, $\theta$ can be plotted against $\ln(r)$ to give a straight line of slope $q / (2\pi k_r Y)$

$$q / (2\pi k_r Y) \quad (15.38)$$

Now, values of wet and dry bulb temperature, together with barometric pressures measured at the two ends $a$ and $b$ of the airway allow the corresponding values of sigma heat, $S$, to be established (Section 14.6). Furthermore, anemometer traverses give the airflow, $Q$, (Section 6.2) and, hence, the mass flow of air, $M$

$$M = Q \rho$$

where $\rho = $ actual density of air (kg/m$^3$)

Then

$$q = M (S_b - S_a) \quad W \quad (15.39)$$

Substituting for $q$ in the expression for the measured slope of the $\theta \ln(r)$ graph (15.38) allows $k_r$ to be determined. An average value of rock thermal conductivity will be given by the data obtained from several boreholes.

Although simple in principle, this in-situ method of measuring effective thermal conductivity of strata may involve several difficulties. First, the airflow in the airway and the psychrometric conditions at the inlet should, ideally, remain constant. Accuracy will be improved if all
measurements are made electronically and data logged every few minutes for several days. The plots of $\theta$ against $\ln(r)$ are not always straight lines. A sudden change in slope along a given set of borehole data indicates that the borehole has intersected a change in rock type. A more gradual curvature suggests that the strata temperatures have not reached equilibrium. Deviations towards the mouth of the hole indicate non-steady conditions in the airway. In such situations, the slope of the line at the rock surface should be used in the determination of $k_r$. Differences in the slopes of the lines are often found between boreholes drilled in different directions from the same cross-section of the airway. Such deviations may indicate differences in rock type or, often, the effect of water distribution within the strata.

A major advantage of this method is that it gives the effective thermal conductivity of the complete envelope of rock around the chosen cross-section. Other techniques have been developed that produce an artificial source of heat within a borehole with instrumentation to monitor the resulting temperature field at other locations along the borehole (Danko, 1987). Such equipment can produce results within a few hours for more limited representative volumes of rock.

15.3. OTHER SOURCES OF HEAT

15.3.1. Autocompression

When air or any other fluid flows downward, some of its potential energy is converted to enthalpy ($H = PV + U$, equation (3.19)) producing increases in pressure, internal energy and, hence, temperature. Actual examples are shown on Figure 8.3.

The rise in temperature as air falls through a downcast shaft or other descentional airway is independent of any frictional effects (Section 3.4.1). Ignoring the small change in kinetic energy, the steady flow energy equation (3.22) gives

$$(H_2 - H_1) = (Z_1 - Z_2)g + q_{12}$$

(15.40)

where

- $H$ = enthalpy J/kg
- $Z$ = height above datum (m)
- $g$ = gravitational acceleration ($m/s^2$)
- $q$ = heat added from surroundings (J/kg)

(subscripts 1 and 2 refer to the inlet and outlet ends of the airway respectively).

This equation shows that the increase in enthalpy is, in fact, due to two components, (a) the heat actually added, $q_{12}$, and (b) the conversion of potential energy $(Z_1 - Z_2)g$.

The effects of autocompression are virtually independent of airflow. In deep mines, the intake air leaving the bottoms of downcast shafts may already be at a temperature that necessitates air cooling. This is the inevitable result of autocompression.

The reverse effect, auto-decompression, occurs in upcast shafts or ascentional airways. This is usually of less concern as upcast air will have no effect on conditions in the workings. However, the reduction in temperature due to auto-decompression in upcast shafts can result in condensation and fogging (Section 9.3.6). The mixture of air and water droplets may then reduce the life of the impellers of exhaust fans sited at the shaft top.

The increase in temperature due to depth is sometimes known as the adiabatic lapse rate. For completely dry airways, the change in enthalpy is given by equation (3.33)
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\[ H_2 - H_1 = C_{pm} (T_2 - T_1) \quad \text{J/kg} \]

where \( T \) = dry bulb temperature (°C or K) and \( C_{pm} \) = specific heat of the actual (moist) air (J/kg°C).

(We are reverting to the symbol \( T \) for temperature in this section in order that we may use it to denote absolute temperatures). Hence, equation (15.40) gives the adiabatic (\( q_{12} = 0 \)) rise in dry bulb temperature to be

\[ (T_2 - T_1) = \frac{(Z_1 - Z_2) g}{C_{pm}} \quad \text{°C} \] (15.41)

Substituting for \( C_{pm} \) from equation (14.16) gives

\[ (T_2 - T_1) = \frac{(1 + X) (Z_1 - Z_2) g}{(1005 + 1884 X) X} \quad \text{°C} \] (15.42)

where \( X \) = mass of water vapour (kg/kg dry air).

For a completely dry shaft or slope and at a value of \( g = 9.81 \) m/s\(^2\), this gives a dry bulb adiabatic temperature lapse rate varying from 0.976 °C per 100 m depth for dry air to 0.952 °C per 100 m depth at a water vapour content of 0.03 kg/kg dry air.

However, in the majority of cases, the shaft or airway surfaces are not completely dry and the rate of increase in dry bulb temperature is eroded by the cooling effect of evaporation. If the increase in vapour content of the air due to evaporation is \( \Delta X \) kg/kg dry air, then this will result in a conversion of sensible heat to latent heat, \( L \Delta X \) (J/kg) where \( L \) = latent heat of evaporation.

Equation (15.41) then becomes

\[ (T_2 - T_1) = \frac{(Z_1 - Z_2) g - L \Delta X}{C_{pm}} \quad \text{°C} \] (15.43)

Using mid-range values of \( C_{pm} = 1010 \) J/kgK and \( L = 2453 \times 10^3 \) J/kg gives the approximation

\[ (T_2 - T_1) = 0.00971 (Z_1 - Z_2) - 2428.7 \Delta X \quad \text{°C} \] (15.44)

This equation is shown graphically in Figure 15.7 and is useful only if the increase in vapour content is known. In practical design exercises, \( \Delta X \) is normally unknown but an estimate may be made of the wetness of the shafts. Recourse must then be made to computer simulation techniques (Chapter 16). It must be emphasized that equations (15.41 to 15.44) refer to the dry bulb temperature only and while the air remains unsaturated. Should saturation occur then the dry bulb temperature will follow the wet bulb temperature adiabatic lapse rate.

The behaviour of the wet bulb temperature due to adiabatic compression is more predictable than dry bulb temperature. The enthalpy (and sigma heat) of the air increase linearly with depth in the absence of any heat transfer with the surroundings. Furthermore, the increase in air pressure is also near linear with respect to depth. Hence, as wet bulb temperature varies only with sigma heat and pressure, the wet bulb adiabatic lapse rate in a shaft will be near constant irrespective of shaft wetness. Any significant deviation in the wet bulb temperature can be due only to heat transfer with the walls or shaft fittings.
Unfortunately, calculation of the wet bulb adiabatic lapse rate is somewhat more involved than for the dry bulb.

We can write the variation in wet bulb temperature, \( T_w \), with respect to depth, \( Z \), as:

\[
\frac{dT_w}{dZ} = \frac{dT_w}{dP} \frac{dP}{dZ} \quad \text{°C} \quad (15.45)
\]

From the steady flow energy equation for isentropic conditions \( (F = q = 0) \) and negligible change in kinetic energy (equation (3.25)),

\[
-g \, dZ = \dot{V} \, dP \quad \text{or} \quad \frac{dP}{dZ} = -\frac{g}{\dot{V}} \quad \text{N/m}^3 \quad \text{or} \quad \text{Pa/m} \quad (15.46)
\]

where \( \dot{V} = \) specific volume \( (\text{m}^3/\text{kg}) = 287.04 \, T_w/(P - 0.378 \, e_d) \) for saturation conditions (equation (14.19))
Furthermore, we have already derived the isentropic pressure/temperature relationship for an air/vapour/liquid water mixture for fans as equation (10.56). The same equation applies to any other isentropic airflow process involving air/vapour/liquid water mixtures. In order to track the adiabatic wet bulb temperature lapse rate, we apply the constraint that critical saturation is maintained throughout the shaft, i.e. $X = X_s$. Equation (10.56) then becomes

$$
\frac{dTw}{dP} = \frac{0.286 \left[ (1 + 1.6078 X_s) \frac{T_w}{P} + \frac{L_w X_s}{287.04 (P - e_s)} \right]}{1 + 1.7921 X_s + \frac{L_w^2 P X_s}{463.81 \times 10^3 (P - e_s) T_w^2}} \quad ^\circ C/\text{Pa} \quad (15.47)
$$

In this equation $T_w$ is the absolute wet bulb temperature (K) and barometric pressure, $P$, is in Pa.

Equations (15.45 to 15.47) allow the wet bulb lapse rate, $dT_w/dZ$ to be determined for any given $P$ and $T_w$ as $X_s$ and $e_s$ are functions of pressure and temperature only (Section 14.6). Tracking the behaviour of $dT_w/dZ$ for a downcast shaft from equation (15.47) and commencing from known shaft top pressure and wet bulb temperature shows that the adiabatic lapse rate for wet bulb temperature is not exactly constant but decreases slowly with depth. For manual application, Figure 15.8 has been constructed from values calculated for a depth of 500 m below the shaft top. This gives an accuracy within 4 per cent for a 1000m shaft over the ranges of pressure and starting temperatures shown.

It should be recalled that the equations derived for autocompression effects give the changes in temperature due to depth only. Actual measurements will reflect, also, heat transfer with the shaft walls or other sources. These are likely to be most noticeable in downcast shafts due to transients in the surface conditions.

15.3.2. Mechanized equipment

The operation of all mechanized equipment results in one, or both, of two effects; work is done against gravity and/or heat is produced. A conveyor transporting material up an incline, a shaft hoist and a pump are examples of equipment that work, primarily, against gravity. Vehicles operating in level airways, rock breaking machinery, transformers, lights and fans are all devices that convert an input power, via a useful effect, into heat.

With the exceptions of compressed air motors and devices such as liquid nitrogen engines, all other forms of power including electricity and chemical fuels produce thermal pollution that must be removed by the environmental control system.

Increasing utilization and power of mechanization in mines and other subsurface facilities has resulted in such equipment joining geothermal effects and autocompression as a major source of heat. With machines consuming about 2 MW of electrical power on some highly mechanized longwall faces, a number of coal mines in the United Kingdom and Europe had to resort to refrigeration equipment at depths below surface where it was not previously required. The calculation of equipment heat is straightforward compared to that for strata heat.

---

1 The error depends primarily on the starting wet bulb temperature and is approximately 4 per cent at an initial wet bulb temperature of 0°C. At a starting wet bulb temperature of 20°C the error is about 2 per cent.
The following algorithm gives a good approximation to these curves:

Enter shaft top wet bulb temperature (tw) °C
Enter shaft top barometric pressure (P) kPa
a = -7.6286E-07*P + 1.7593E-04
b = -1.2554E-07*P^2 + 2.238E-05*P + 1.1012E-02
c = -9.4107E-06*P^2 + 3.8054E-03*P + 0.33327
Wet bulb temp. adiabatic lapse rate = a*tw^2 - b*tw + c
15.3.2.1. Electrical equipment

Figure 15.9 illustrates the manner in which the power taken by an electrical machine is utilized. The machine efficiency is relevant, within this context, in two ways. First, the total amount of heat produced can be reduced only if the machine is replaced by another of greater efficiency to give the same mechanical power output at lower power consumption. For any given machine, the total heat produced is simply the rate at which power is supplied, less any work done against gravity. Secondly, the efficiency of the machine determines the distribution of the heat produced. The higher the efficiency, the lower the heat produced at the motor and transmission, and the greater is the percentage of heat produced at the pick-point, conveyor rollers, along the machine run or by any other frictional effects caused by the operation of the device.

Figure 15.9  Heat produced by electrical machines.
Example.
A 2000 m long conveyor transports 500 t/hr through a vertical lift of 200 m. If the conveyor motor consumes 1000 kW at a combined motor/transmission efficiency of 90%, calculate the heat emitted

(1) at the gearhead and
(2) along the length of the conveyor.

Solution.
Work done against gravity = mass flow \times g \times vertical lift

\[
\text{Work done against gravity} = \frac{500 \times 1000}{60 \times 60} \times 9.81 \times 200 = 272.5 \times 10^3 \text{ W or } 272.5 \text{ kW}
\]

1. Heat generated at gearhead = (100 - 90)% of 1000 kW = 100 kW
2. Heat generated along length of conveyor = 1000 - 272.5 - 100 = 627.5 kW

15.3.2.2. Diesel equipment

The internal combustion engines of diesel equipment have an overall efficiency only about one third of that achieved by electrical units. Hence, diesels will produce approximately three times as much heat as electrical equipment for the same mechanical work output. This can be demonstrated by taking a typical rate of fuel consumption to be 0.3 litres per rated kW per hour. At a calorific value of 34 000 kJ/litre for the diesel fuel, the heat produced is

\[
= \frac{0.3}{60 \times 60} \times \frac{(\text{litres})}{(\text{kW output. second})} \times 34000 \frac{(\text{kJ heat})}{(\text{litre})}
\]

= 2.83 kJ/s (or kilowatts) of heat emitted for each kilowatt of mechanical output.

This heat appears in three ways each of which may be of roughly the same magnitude. One third appears as heat from the radiator and machine body, one third as heat in the exhaust gases and the remaining third as useful shaft power which is then converted to heat (less work done against gravity) by frictioanal processes as the machine performs its task.

As with other types of heat emitting equipment there is little need, in most cases, to consider peak loads. It is sufficient to base design calculations on an average rate of machine utilization. The most accurate method of predicting the heat load is on the basis of average fuel consumption during a shift. However, in many mines, records of fuel consumption by individual machines or, even, in separate sections of the mine, seem not to be maintained. The ventilation planner often must resort to the type of calculation shown above and using an estimated value for machine utilization. The latter is defined as the fraction of full load which, if maintained continuously, would use the same amount of fuel as the actual intermittent load on the machine. (Section 16.2.3)

A difference between diesel and electrical equipment is that the former produces part of its heat output in the form of latent heat. Each litre of diesel fuel that is consumed produces approximately 1.1 litres of water (liquid equivalent) in the exhaust gases (Kibble, 1978). This may be multiplied several times over by the evaporation of water from engine cooling systems, some emission control units and enhanced evaporation from airway surfaces. In-situ tests have shown that the factor can vary from 3 to 10 litres of water per litre of fuel (McPherson, 1986) depending upon the power and design of the engine, exhaust treatment system and the proficiency of maintenance.
Example.
Two load-haul-dump vehicles consume 600 litres of diesel fuel in an 8 hour shift. Tests have shown that water vapour is produced at a rate of 5 litres (liquid equivalent) per litre of fuel. If the combustion efficiency is 95 per cent and the total calorific value of the fuel is 34 000 kJ/litre, calculate the sensible and latent heat loads on the stope ventilation system.

Solution.
Total amount of heat produced from burning 600 x 0.95 litres of fuel
\[
= 600 \times 0.95 \times 34 \, 000 \\
= 19.38 \times 10^6 \, \text{kJ in 8 hours} \\
= \frac{19.38 \times 10^6}{8 \times 60 \times 60} \approx 673 \, \text{kW}
\]
(This is equivalent to the continuous running of diesel machinery of rated output = 673/2.83 = 238 kW)

Amount of water produced as vapour
\[
= 600 \times 0.95 \times 5 = 2850 \text{ litres (liquid equivalent)} \\
i.e. \, 2850 \text{ kg of water}
\]

Latent heat emitted in 8 hours
\[
= 2450 \times 2850 = 6.982 \times 10^6 \, \text{kJ}
\]
where 2450 kJ/kg is an average value for the latent heat of evaporation of water.

\[
i.e. \, \frac{6.982 \times 10^6}{8 \times 3600} = 242 \, \text{kJ/s or kW}
\]

Then, sensible heat produced = total heat - latent heat
\[
= 673 - 242 = 431 \, \text{kW}
\]

In summary, operation of the diesels results in heat emissions at an average rate of
- 431 kW sensible heat
- 242 kW latent heat
- 673 kW total heat

In addition to the particulate and gaseous pollutants emitted in diesel exhausts, the heat produced by these machines mitigates against their use in hot mines. Nevertheless, the flexibility and reliability of diesel units has tended to perpetuate their continued widespread employment underground.

15.3.2.3. Compressed air

When compressed air is used for drilling or any other purpose then there are two opposing effects that govern the heat load. First, the work output of the machine will result in frictional heat at the pick point or other places as the machine performs its task. Second, the removal of energy from the compressed air will result in a reduction of the temperature of that air at the exhaust ports of the machine.

If we assume that the change in potential energy of the air is negligible and that there is no significant heat transfer across the machine casing then the steady flow energy for the compressed air motor is
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\[
\frac{u_1^2 - u_2^2}{2} + W = C_p (\theta_2 - \theta_1) \quad \text{J/kg} \quad (15.49)
\]

where \( u_1 \) and \( u_2 \) are the velocities of the air in the supply pipe and the exhaust ports, respectively, \( W \) is the mechanical work done on the air (this is numerically negative as mechanical energy is leaving the system; J/kg), \( C_p \) is the specific heat of air, (1005 J/kg°C) for dry air, \( \theta_1 \) is the temperature of the supply air (°C) and \( \theta_2 \) is the temperature of air at the outlet ports (°C).

In the majority of cases the temperature of the compressed air supplied to the motor is equal to the ambient dry bulb temperature, \( \theta_{amb} \). Furthermore, the velocity of the compressed air in the supply pipe, \( u_1 \), is small compared with that at the exhaust ports, \( u_2 \).

Hence,

\[
\frac{-u_2^2}{2} + W = C_p (\theta_2 - \theta_{amb}) \quad \text{J/kg} \quad (15.50)
\]

Now consider the subsequent process during which the cold exhaust air mixes with the ambient air, increasing in temperature, extracting heat from the general ventilating airstream and, hence, cooling that airstream. Again, the velocity of the ambient airstream will be small compared with that at the outlet ports, \( u_2 \). The steady flow energy equation for this process becomes

\[
\frac{u_2^2}{2} = C_p (\theta_{amb} - \theta_2) - q \quad \text{J/kg} \quad (15.51)
\]

where \( q \) is the heat added to the machine exhaust from the ambient air.

The overall effect may be quantified by adding equations (15.50 and 15.51) giving

\[
W = -q \quad \text{J/kg}
\]

Hence, the mechanical work output of the machine (which is subsequently transferred by friction into heat) is balanced precisely by the cooling effect on the ventilating airstream. If the moisture content of the compressed air is similar to that of the ambient airstream, then there is no net heating or cooling arising from the expansion of the air through the compressed air motor.

Despite this analysis, a cooling effect is certainly noticeable when standing immediately downstream from a compressed air motor. There are two reasons for this. First, the cooling of the ambient airstream, \( q \), by a motor exhausting at sub-zero temperatures is immediate and direct, whilst part of the balancing heat produced by friction may be stored temporarily within the body of the machine, in the solid or broken rock, and distributed over the working length of the machine. Secondly, the compressed air supply will normally be drier and have a much lower sigma heat than the ventilating air. The increase in sigma heat of the exhaust air will then exceed the numerical equivalent of the work output and, hence, result in a true net cooling of the ambient air. This will be of consequence only if the mass flow of compressed air is significant compared with the mass flow of the ventilating airstream. The dryness of compressed air, coupled with a very local kinetic energy effect \( \frac{u_2^2}{2} = C_p(\theta_2 - \theta_1) \) also explains why air issuing from a leaky compressed air pipe appears cool even though no energy has been extracted from it.

In calculating the heat load for a complete airway, district or mine, it errs on the side of safety to assume that there is no net cooling effect from compressed air motors. Only if a detailed analysis in the immediate vicinity of compressed air devices is required need the local cooling effects be considered.
Air compressors are large sources of heat. Indeed, the local cooling effect at the outlet ports of the compressed air motors are reflected by an equivalent amount of heat produced at the compressors, even if the latter are 100 % efficient. This need be of little import if the compressors are sited on surface and the compressed air is cooled before entering the shaft pipes. If, however, compressors are sited in intake airways close to hot workings, then it is beneficial to investigate the circulation of hot water from the compressor coolers through heat exchangers in return airways.

15.3.3. Fissure water and channel flow

Groundwater migrating through strata toward a subsurface opening can add very considerably to the transfer of geothermal heat through the rock (Section 15.2.10). Such water may continue to add heat to the ventilating airstream after it has entered the mined opening.

Fissure water is often emitted at a temperature close to the virgin rock temperature \( (VRT) \). In circumstances of local geothermal activity or radioactive decay, it may even be higher. The total heat load on the mine environment can be calculated from the flowrate and the drop in temperature of the water between the points of emission and effective exit from the mine ventilation system.

Example.
A mine produces 5 million litres of water per day, emitted at an average temperature of 42 °C. When the water is delivered into the shaft sump for pumping to the surface, the water temperature is 32 °C. Determine the heat load from the water on the mine ventilation system.

Solution.

\[
\text{Heat load} = \text{Mass flow} \times \text{specific heat} \times \text{drop in temperature}
\]

\[
= \frac{5000000}{24 \times 60 \times 60} \times \frac{4187}{1000} \times (42 - 32) = 2423 \text{ kW}
\]

The rate at which heat is emitted into the airstream depends upon the difference between the temperature of the air and the water, and whether the water is piped or in open channels. In the latter case, cooling by evaporation will be the major mode of heat transfer and will continue while the temperature of the water exceeds the wet bulb temperature of the air. Hot fissure water should be allowed no more than minimum direct contact with intake air. The most effective means of dealing with this problem are

(a) to transport the water in closed pipes and
(b) to restrict water flow routes to return airways.

A less effective but expedient measure close to the points of water emission is to restrict air/water contact by covering drainage channels by boarding or other materials.

Where chilled service water is employed, condensate run-off may be at a temperature below that of the local wet bulb temperature. While this continues, the condensate will continue to absorb both sensible and latent heat, thus providing a cooling effect on the airflow.

In, or close to the working areas of many mines, it is often inevitable that open drainage channels will be used. Similarly, the footwalls or floors may have areas that are covered with standing or slowly moving water.
The transfer of sensible and latent heat between the surface of the water and the air may be determined from equations (15.34 and 15.35).

\[
q_{\text{sen}} = h_c (\theta_{ws} - \theta_d) \quad \text{W/m}^2
\]
and
\[
q_L = 0.0007 h_L L_{ws} (e_{ws} - e) \quad \text{W/m}^2
\]

where subscript \(ws\) means water surface and \(d\) means dry bulb.

The heat transfer coefficient for the water surface, \(h_c\), is the least certain of the variables, depending upon the geometry of the airway, the condition of the airflow (in particular, the local air velocity) and the degree of turbulence existing in the flowing water. An approximation may be obtained from equations (15.22 and 15.23) using the Reynolds' Number for the airway and a value of coefficient of friction, \(f\), that represents the degree of water turbulence on the liquid surface. This may vary from 0.002 for a calm flat surface to 0.02 for a highly turbulent surface.

Example.
Water flows at a rate of 60 litres per second along an open drainage channel that is 0.5 m wide and 0.25 m deep. The channel is located in an airway of cross-sectional area 12 m² and perimeter 14 m, and which passes an airflow of 50 m³/s. Over a 10 m length of channel, the following mean values are given:

- Water temperature, \(\theta_{ws} = 32 \, ^\circ\text{C}\)
- Air dry bulb temperature, \(\theta_d = 30 \, ^\circ\text{C}\)
- Air wet bulb temperature, \(\theta_w = 25 \, ^\circ\text{C}\)
- Barometric pressure, \(P = 112 \, \text{kPa}\)

Determine the sensible and latent heat emitted from the water to the air and the corresponding drop in water temperature over the 10 m length. Assume that no heat is added to the water from the strata.

Solution.
(a) Determine the heat transfer coefficient for the water surface.

Volume flow of water = \(\frac{60}{1000} = 0.06 \, \text{m}^3/\text{s}\)

Velocity of water = \(\frac{0.06}{0.5 \times 0.25} = 0.48 \, \text{m/s}\)

At this velocity, it is estimated that ripples on the water surface will have an equivalent coefficient of friction, \(f = 0.008\).

The Reynolds' Number for the airway

\[
Re = \frac{Q}{\text{per} \times d} = \frac{268000 \times 50}{14} = 957140
\]

Equation (15.22) gives the Nusselt Number to be \(N_u = 2127\) (this can be estimated directly from Figure 15.5)

Hydraulic mean diameter of the airway,

\[
d = \frac{4A}{\text{per} \times 14} = 3.429 \, \text{m}
\]
The convective heat transfer coefficient is then given by equation (15.23).
Using $k_a = 0.026 \text{ W/(m°C)}$ (reference Table 15.1),

$$h_c = \frac{2127 \times 0.026}{3.429} = 16.1 \text{ W/(m}^2\text{°C)}$$

(b) Determine the psychrometric conditions.
Using the psychrometric relationships given in Section 14.6, the following values are determined:

for the water surface at 32 °C

\[ L_{ws} = 2426 \times 10^3 \text{ J/kg} \]

and

\[ e_{ws} = 4753 \text{ Pa} \]

for the air at $\theta_w = 25 ^\circ \text{ C}$, $\theta_d = 30 ^\circ \text{ C}$ and $P = 112 \text{ kPa}$,

\[ e = 2805 \text{ Pa} \]

[The advantages of keeping the psychrometric equations of Section 14.6 programmed into a pocket calculator should be very obvious by now.]

(c) Determine the heat transfers from the water to the air:
Sensible heat,

\[ q_{sen} = h_c (\theta_{ws} - \theta_d) = 16.1 (32 - 30) = 32.2 \text{ W/m}^2 \]

Latent heat,

\[ q_L = 0.0007 h_c L_{ws} \frac{(e_{ws} - e)}{P} = 0.0007 \times 16.1 \times 2426 \times 10^3 \times \frac{4753 - 2805}{112000} = 475.5 \text{ W/m}^2 \]

As the width of the drainage channel in the 10 m length is 0.5 m, the heat flows may be stated as:

Sensible heat

\[ \frac{32.2 \times 10 \times 0.5}{1000} = 0.161 \text{ kW} \]

Latent heat

\[ \frac{475.5 \times 10 \times 0.5}{1000} = 2.378 \text{ kW} \]

These values illustrate the dominant effect of latent heat transfer and why it is important to prevent direct contact between the air and the water.

The total heat gained by the air from the water is the sum of the sensible and latent heat transfers,

\[ q = 2.378 + 0.161 = 2.539 \text{ kW} \]

(d) Determine the drop in temperature of the water.
As 1 litre of water has a mass of 1 kg, the mass flow of water, $m = 60 \text{ kg/s}$. The total heat lost by the water, then becomes

\[ q = m \Delta \theta C_w \]

where

\[ \Delta \theta = \text{change in water temperature (°C)} \]

and

\[ C_w = \text{specific heat of water (4187 J/(kg°C))} \]

giving

\[ \frac{\Delta \theta}{m C_w} = \frac{2.539 \times 10^3}{60 \times 4187} = 0.0101 \text{ °C in the 10 m length.} \]
15.3.4. Oxidation

Coal and sulphide ore mines are particularly liable to suffer from a heat load arising from oxidation of fractured rock. An estimate of the heat produced can be determined from the rate of oxygen depletion.

Assuming complete combustion of carbon and using the atomic weights of the elements,

\[
\begin{align*}
C + O_2 & \rightarrow CO_2 \\
12 \text{ kg} + 32 \text{ kg} & \rightarrow 44 \text{ kg}
\end{align*}
\]  
(15.52)

Each 1 kg of oxygen consumed oxidizes 12/32 kg of carbon. Taking the calorific value of carbon as 33 800 kJ/kg gives a corresponding heat production of \((12/32) \times 33 \text{,}800 = 12 \text{,}675 \text{ kJ}\) of heat per kg of oxygen used.

Similarly, for complete oxidation of sulphur:

\[
\begin{align*}
S + O_2 & \rightarrow SO_2 \\
32 \text{ kg} + 32 \text{ kg} & \rightarrow 64 \text{ kg}
\end{align*}
\]  
(15.53)

Each 1 kg of oxygen consumed will oxidize 1 kg of sulphur (calorific value 9304 kJ/kg) to produce 9304 kJ of heat.

Example.

10 m^3/s of air enters a working district in a coal mine at an oxygen content of 21% by volume, and leaves at 20.8%. Calculate the heat generated by oxidation assuming complete combustion.

Solution.

The equivalent volume flow of oxygen at entry = 0.21 \times 10 = 2.1 m^3/s

The equivalent volume flow of oxygen at exit = 0.208 \times 10 = 2.08 m^3/s

Taking the density of oxygen to be 1.3 kg/m^3 gives the oxygen depletion rate to be

\[
(2.1 - 2.08) \times 1.3 = 0.026 \text{ kg/s}
\]

Heat produced = 0.026 \times 12 \text{,}675 = 330 \text{ kW}

The type of calculation illustrated by this example is useful in determining the heat produced by oxidation in an existing mine or section of a mine where air samples can be taken for analysis in intake and return airways. The extent to which oxidation takes place depends upon the mineralogical content of the material being oxidized, the psychrometric condition of the air and the surface area exposed. These factors make it difficult to predict heat loads from oxidation by other than empirical means.

In the case of open surfaces in operating airways and working places, the heat of oxidation will be an immediate and direct load on the mine ventilation system. On the other hand the heat that is produced by oxidation in waste areas, old workings, orepasses or in caved stopes will initially be removed only partially by leakage air. The remainder will be retained causing a rise in the temperature of the rock. This, in turn, enhances the rate of heat transfer to the leakage air and, in most cases, an equilibrium is reached when the heat removed by the air balances the heat of oxidation. However, in minerals liable to spontaneous combustion the increased temperature of the rock will accelerate the oxidation process to an extent that the heat removed may not reach equilibrium with the heat produced. In such cases the temperature will continue to rise until the rock becomes incandescent resulting in a spontaneous fire (Section 21.4).
15.3.5. Explosives

The heat that is produced during blasting varies with the type and amount of explosives used (charge density). The amount of heat released by most explosives employed in mining falls within the range of 3700 kJ/kg for ANFO\(^2\) to 5800 kJ/kg for nitro-glycerine (Whillier, 1981). This heat is dispersed in two ways. First, a fraction of it will appear in the blasting fumes and cause a peak heat load on the ventilation system following blasting. In mines where a re-entry period is enforced this peak load should have cleared prior to personnel being readmitted to the area.

Secondly, the remainder of the heat will be stored in the broken rock. The magnitude of this will depend upon the mining method. If the rock is blasted into a free space through which the ventilating airstream passes, such as an open stope or on a longwall face, and the fragmentation is high, then as much as 40 - 50 % of the heat produced by the explosive may be removed rapidly as a peak load with the blasting fumes. On the other hand, if the blast occurs within a region through which there is little or no ventilation such as in sublevel or forced caving techniques, or if the fragmentation is poor, then a much larger proportion of the heat will be retained in the rock.

Example.

In a 2000 tonne blast, the charge density of ANFO is 0.8 kg/t. It is estimated that 20% of the blast heat will be removed within 1 h with the blasting fumes.

(1) Calculate the mean value of the rate of heat removed by the airflow during this hour.

(2) If the specific heat of the rock is 950 J/kg °C determine the average increase in temperature of the rock due to blasting.

Solution:

1. Mass of explosive used   =   2000 x  0.8     =   1 600 kg ANFO
   Heat produced by ANFO =   3700 x 1600   =   5 920 000 kJ
   
Twenty per cent of this heat is removed in the blasting fumes over 1 hour.

   Average rate of heat removal with blasting fumes
   
   \[
   \text{Average rate} = \frac{5 920 000}{3600} \times 0.2 \ \text{kJ/s} = 329 \text{ kW}
   \]

   2. Heat retained in rock   =   5 920 000 x 0.8   =   4 736 000 kJ

   Rise in rock temperature   =   \frac{\text{Heat retained}}{\text{Mass} \times \text{Specific heat}} = \frac{4 736 000}{2000 \times 1000 \times 0.950} = 2.49 °C

15.3.6. Falling rock

When strata or fractured rock moves downward under gravitational effects, the loss of potential energy will ultimately produce an increase in temperature through fragmentation, impact, braking, or other frictional effects. The fraction of the resulting heat that produces a load on the ventilation system again depends upon the exposure of the broken rock to a ventilating airstream. Hence, for example, a large amount of heat is produced by the mass of superincumbent strata subsiding through to the surface over a period of time. Fortunately, the vast majority of this is retained within the rock mass, raising its temperature very slightly, and little enters the mine ventilation system.

---

\(^2\) Ammonium nitrate, fuel oil mixture.
On the other hand, mineral descending through an ore pass or vertical bunker may immediately, or subsequently, be exposed to a ventilating airstream. The loss of potential energy of the rock will then appear as a heat load on the ventilation system. This can be significant.

Assuming that there is negligible heat transfer between the fragmented material and the surrounding strata, the rise in temperature of falling rock, $\Delta \theta_r$, is given by the expression

$$\Delta \theta_r = \frac{Az g}{C} \quad ^\circ C \quad (15.54)$$

where $\Delta z$ is the distance fallen (m),
$g$ is the gravitational acceleration (9.81 m/s$^2$),
$C$ is the specific heat of rock (J/kg°C)

or

$$\Delta \theta_r = \frac{981}{C} \quad ^\circ C \text{ per } 100 \text{ m fall} \quad (15.55)$$

15.3.7. Fragmented rock

When fragmented rock is exposed to a ventilating airstream and there is a temperature difference between the rock and the air, then heat transfer will take place. This will occur on working faces, drawpoints, conveyors and along other elements of a mineral transportation system.

The heat load from broken rock is given by

$$mC(\theta_1 - \theta_2) \quad \text{kW} \quad (15.56)$$

where

- $m$ is the mass flow of rock (kg/s),
- $C$ is the specific heat of rock (kJ/kg°C),
- $\theta_1$ is the temperature of the broken rock immediately after fragmentation ($^\circ C$) and
- $\theta_2$ is the temperature of the rock at exit from the ventilation system ($^\circ C$).

In most mining methods the temperature of the solid rock immediately prior to fragmentation will be less than the virgin rock temperature (VRTX) due to cooling of the rock surface. On the other hand, the process of fragmentation, whether by blasting or mechanized techniques, will raise the temperature of the broken rock. As shown in earlier sections, estimates may be made of these effects. However, in practice, the procedure may be simplified by assuming that the temperature of the newly broken rock, $\theta_1$, is equal to VRT.

The mean temperature of the broken rock at exit from the system, $\theta_2$, will depend upon the degree of fragmentation, the exposure of rock surfaces to the airstream, and the velocity and psychrometric condition of the air. Mineral transported along a conveyor system will cool to a much greater extent than in a locomotive system. Furthermore, wetted material will yield up its heat more readily due to evaporative cooling. Although the rate of heat transfer from a given particle size of rock to a known quantity and quality of airflow can be calculated, the many variables in any actual transportation system enforce the use of empirical measurements of temperature along the mineral transportation system. Such measurements are facilitated by the use of infra-red thermometry.
Example.
Fragmented ore of specific heat 900 J/(kg°C) enters the top of an ore pass at a temperature of 35°C and at a rate of 500 tonnes per hour. At an elevation 200 m below, the ore is discharged onto a conveyor and reaches the shaft bottom at a temperature of 32°C. Calculate the heat transferred from the broken ore to the ventilation system over the length of the conveyor.

Solution.
From equation (15.54), temperature rise of the rock in the ore pass

\[
\Delta T = \frac{200 \times 9.81}{900} = 2.18 \, ^\circ C
\]

(assuming no heat transfer between the fragmented ore and the surrounding strata)

Temperature of rock at bottom of ore pass = 35 + 2.18 = 37.18 °C

Mass flow of ore, \( m = \frac{500 \times 1000}{3600} \) = 138.9 kg/s

Heat transfer to the airflow from ore on the conveyor (see equation (15.56))

\[
mC(\theta_1 - \theta_2) = 138.9 \times 0.900 \times (37.18 - 32) = 647 \, kW
\]

15.3.8. Metabolic heat

The rate at which the human body produces metabolic heat depends upon a number of factors including rate of manual work, physical fitness and level of mental stress. The question of heat transfer between the human body and the surrounding environment is of great importance in ascertaining the risk of heat stress in mine workers (Chapter 17). However, metabolic heat makes only a small contribution to mine heat load. It may, nevertheless, become significant where labour intensive activities take place in a location of limited throughflow ventilation such as a poorly ventilated heading or a barricaded refuge chamber.

Like other heat engines, the human body emits heat by three mechanisms. The most significant of these in physiological processes is heat loss from the body surface. Secondly, the large, wet surface area of the lungs provides an effective heat exchanger and ‘exhaust’ heat is emitted through respiration. Third, any mechanical work done by the individual on the external surroundings will produce frictional heat, unless that work is done against gravity. As the human body is an inefficient heat engine, this latter mechanism is the least significant and is often omitted in calculations of physiological heat transfer.

The metabolic heat produced by a fit worker who is acclimatized to the environment varies from about 100 W for sedentary work, through 400 W for a medium level of activity such as walking, to over 600 W during intermittent periods of strenuous manual work.(See Table 17.1)
BIBLIOGRAPHY


APPENDIX A15.1

Analytical solution of the three dimensional transient heat conduction equation (15.13) as obtained by Carslaw and Jaeger (1956).

\[ G = 4 \pi^2 \int_0^\infty \frac{e^{-V^2\tau}}{I_0(V) + V / DL_1(V)^2 + [Y_0(V) + V / DY_1(V)]^2} dV \]

where \( I_0, I_1, Y_0 \) and \( Y_1 \) are Bessel functions and \( V \) is the variable of integration.

APPENDIX A15.2

Gibson’s algorithm for the numeric determination of dimensionless temperature gradient, \( G \).

Enter

\( \alpha \) (rock diffusivity, \( m^2/s \))
\( t \) (age of airway, seconds)
\( r_a \) (effective radius of airway = perimeter/(2\( \pi \)), metres)
\( k \) (thermal conductivity of rock, \( W/(m^\circ C) \)).

Then

\( F = \alpha t / r_a^2 \) (Fourier Number)
\( B = hr_a / k \) (Biot Number)
\( x = \log_{10}(F) \)
\( y = \log_{10}(B) \)
\( c = x(0.000104x + 0.000997) - 0.001419 \)
\( c = - \{ x[x(xc - 0.46223) + 0.315553] + 0.006003 \} \)
\( d = y - (x(4x - 34) - 5)/120 \)
\( d = 0.949 + 0.1 \exp(-2.69035d^2) \)
\( m = \sqrt{(y - c)^2 + \left( \frac{216 + 5x}{70} \right) \left( \frac{0.0725 + 0.01 \tan^{-1} \left( \frac{x}{0.7048} \right)}{70} \right)} \)
\( n = (y + c - m)/2 \)
\( G = 10^n / d \)
APPENDIX A15.3.

Background to equations for the heat transfer coefficient, \( h \).

Consider the transfer of heat, \( q \) (W/m\(^2\)) across the boundary layers shown on Figure A15.1. In the laminar sublayer, there are no cross velocities and Fourier's law of heat conduction applies:

\[
q = -k_a \frac{d\theta}{dy} \quad \text{W/m}^2 \quad (A15.1)
\]

where

- \( k_a \) = thermal conductivity of air W/(m°C)
- \( \theta \) = fluid temperature (°C) and
- \( y \) = distance from the wall (m)

In the turbulent boundary layer (and within the mainstream), heat transfer is assisted by eddy action and the equation becomes:

\[
q = -(k_a + \rho_a C_p E_h) \frac{d\theta}{dy} \quad \text{W/m}^2 \quad (A15.2)
\]

where

- \( \rho_a \) = air density (kg/m\(^3\))
- \( C_p \) = specific heat (1005 J/kg°C for dry air) and
- \( E_h \) = eddy diffusivity of heat (m\(^2\)/s)

The term \( \rho_a C_p \) (J/m\(^3\)°C) is the amount of heat transported in each m\(^3\) for each Centigrade degree of temperature difference while \( E_h \) (m\(^2\)/s) represents the rate at which this heat is transported by eddy action. The product \( \rho_a C_p E_h \) is much larger than \( k_a \). 
Equation (A15.2) can be re-written as

\[
\frac{q}{\rho_a C_p} = -\left( \frac{k_a}{\rho_a C_p} + E_h \right) \frac{d\theta}{dy} \quad \text{m}^2 \text{C} \text{ s}^{-1}
\]

\[
= -\left( \alpha + E_h \right) \frac{d\theta}{dy} \quad \text{(A15.2a)}
\]

as \( \alpha = \frac{k_a}{\rho_a C_p} \) = thermal diffusivity of still air (m\(^2\)/s)

Figure A15.1 illustrates two boundary layers only. In fact, a third transitional or buffer layer exists between the laminar and turbulent layers, within which both conduction and eddy diffusion are significant. This is similar to the transitional region between laminar flow and fully developed turbulence shown on Figure 2.6 for pipes.

Let us now consider the transfer of momentum across the boundary layers. For the laminar sublayer where the flow is viscous, Newton’s equation (2.22) applies:

\[
\tau = \mu \frac{du}{dy} \quad \text{N/m}^2 \quad \text{(A15.3)}
\]

where \( \tau \) = shear stress transmitted across each lamina of fluid (N/m\(^2\) or Pa)

\( \mu \) = coefficient of dynamic viscosity Ns/m\(^2\)

\( u \) = fluid velocity (m/s)

Note that the units of shear stress may be written also as Ns/(m\(^2\)s)

Therefore, as Ns are the units of momentum, \( \tau \) also represents the transfer of momentum across each square metre per second.

For the turbulent boundary layer, the equation becomes

\[
\tau = (\mu + \rho_a E_m) \frac{du}{dy} \quad \text{N/m}^2 \quad \text{(A15.4)}
\]

where \( E_m \) = eddy diffusivity of momentum (m\(^2\)/s) and

\( \rho_a E_m \) = momentum transfer across each m\(^2\) by eddy action (Ns/m\(^2\))

Again, \( \rho_a E_m \) is much larger than \( \mu \).

Then

\[
\frac{\tau}{\rho_a} = \left( \frac{\mu}{\rho_a} + E_m \right) \frac{du}{dy}
\]

\[
= \left( \nu + E_m \right) \frac{du}{dy} \quad \frac{J}{\text{kg}} \quad \text{(A15.5)}
\]

where \( \nu = \frac{\mu}{\rho_a} \) = kinematic viscosity or momentum diffusivity (m\(^2\)/s)
In order to combine heat transfer and momentum transfer, Reynolds divided equations (A15.2a) and (A15.5), giving,

\[
\frac{q}{\rho_a C_p \tau} = -\frac{(\alpha + E_h)}{(v + E_m)} \frac{d\theta}{dy} dy du
\]

or

\[
\frac{q}{C_p \tau} = -\frac{(\alpha + E_h)}{(v + E_m)} du
\]  \hspace{1cm} (A15.5a)

Reynolds argued that as the eddy components predominate and the eddy diffusivity for heat, \(E_h\), must be closely allied to the eddy diffusivity for momentum, \(E_m\), we may equate those terms, leaving

\[
\frac{q}{C_p \tau} = -\frac{d\theta}{du}
\]  \hspace{1cm} (A15.6)

Reynolds integrated this equation directly, making the approximation that \(u\) remained linear with respect to \(y\) across the composite boundary layers (see the velocity profile on Figure A15.1). However, both Taylor and Prandtl, working independently, later realized that the integration had to be carried out separately for the laminar sublayer and the turbulent boundary layer.

**Laminar sublayer**

From equation (A15.1)

\[
q \int_{\delta}^{\infty} dy = -k_a \int_{\theta_s}^{\theta_b} d\theta
\]

\[
q \delta = k_a (\theta_s - \theta_b)
\]  \hspace{1cm} (A15.7)

where

\(\delta\) = thickness of laminar sublayer (m)

\(\theta_s\) = temperature at the surface (°C)

and

\(\theta_b\) = temperature at the edge of the laminar sublayer (°C)

Similarly, integrating equation (A15.3) from zero velocity at the wall to \(u_b\) at the edge of the laminar sublayer

\[
\tau = \mu \frac{u_b}{\delta}
\]  \hspace{1cm} (A15.8)

Dividing (A15.7) by (A15.8) gives

\[
\frac{q}{\tau} = k_a \frac{(\theta_s - \theta_b)}{\mu u_b}
\]  \hspace{1cm} (A15.9)

**Turbulent boundary layer**

Here we employ equation (A15.6)

\[
\frac{q}{C_p \tau} du = -d\theta
\]

and integrate from the edge of the laminar sublayer (subscript \(b\)) to the mainstream (subscript \(m\)) (see Figure A15.1):
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\[ \frac{q}{C_p \tau} \int_{u_b}^{u_m} du = - \int_{u_b}^{u_m} d \theta \]

giving \[ \frac{q}{C_p \tau} = \frac{\left( \theta_b - \theta_m \right)}{\left( u_m - u_b \right)} \] (A15.10)

A difficulty here is that neither \( \theta_b \) nor \( u_b \) are amenable to accurate location or measurement. We can eliminate \( \theta_b \) by rewriting (A15.9) as

\[ \theta_b = \theta_s - \frac{q}{\tau k_a} u_b \] (A15.11)

Substituting for \( \theta_b \) in equation (A15.10) and engaging in some algebraic manipulation leads to

\[ \frac{q}{C_p \tau} = \frac{\left( \theta_s - \theta_m \right)}{u_m} \left[ \frac{1}{1 + \frac{u_b}{u_m} \left( \frac{\mu C_p}{k_a} - 1 \right)} \right] \]

Now the combination \( \mu C_p/k_a \) is another dimensionless number known as the Prandtl Number \( P_r \)

\[ \frac{q}{C_p \tau} = \frac{\left( \theta_s - \theta_m \right)}{u_m} \left[ \frac{1}{1 + \frac{u_b}{u_m} \left( P_r - 1 \right)} \right] \] (A15.12)

Furthermore, the shear stress, \( \tau \), is related to the coefficient of friction, \( f \), and the mainstream velocity, \( u_m \), by equation (2.41)

\[ \tau = f \rho_a \frac{u_m^2}{2} \]

Also,

\[ q = h(\theta_s - \theta_m) \] from equation (15.16) where
\[ \theta_m = \theta_d = \text{dry bulb temperature of the mainstream} \]

Substituting for \( \tau \) and \( q \) in equation (A15.12),

\[ \frac{h(\theta_s - \theta_m)}{C_p f \rho_a u_m^2} = \frac{\left( \theta_s - \theta_m \right)}{u_m} \left[ \frac{1}{1 + \left( P_r - 1 \right) \frac{u_b}{u_m}} \right] \]

\[ \frac{h}{C_p \rho_a u_m} = \frac{f}{2} \left[ \frac{1}{1 + \left( P_r - 1 \right) \frac{u_b}{u_m}} \right] \]

The left hand side of this equation can be separated into three dimensionless groups

\[ \frac{h d}{k_a} \frac{\mu}{\rho_a u_m d} \frac{k_a}{C_p \mu} = \frac{N_u}{R_e} \frac{1}{P_r} \]

Nusselt No. \( N_u \) \hspace{1cm} Reynolds No. (Re) \hspace{1cm} Prandtl No. \( P_r \)
Hence

$$N_u = \frac{f}{2} \text{Re} \frac{P_r}{1 + \frac{1.5}{\text{Re}^{1/8} P_r^{1/6} \left( P_r \frac{f}{f_0} - 1 \right)}}$$  \hspace{1cm} (A15.13)

This is sometimes known as the Taylor or Taylor-Prandtl equation and is the basic relationship that has been used by numerous other workers. These varied in the manner in which they treated the $u_b/u_m$ ratio as $u_b$ remains elusive to measure. Taylor himself used an empirical value of $u_b/u_m = 0.56$. Several other relationships are listed in Table A15.1. A more comprehensive listing is given by Danko (1988)

<table>
<thead>
<tr>
<th>Authority</th>
<th>Value of $u_b/u_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Taylor</td>
<td>0.56</td>
</tr>
<tr>
<td>Von Kármán</td>
<td>$9.77\sqrt{f/2}$</td>
</tr>
<tr>
<td>Rogers and Mayhew</td>
<td>$1.99\text{Re}^{-0.125}$</td>
</tr>
</tbody>
</table>

Table A15.1 Some reported values of the $u_b/u_m$ ratio

A comprehensive theoretical and experimental study of flow over rough surfaces by Nunner (1956) produced a more sophisticated equation

$$N_u = \frac{f}{2} \text{Re} \frac{P_r}{1 + \frac{1.5}{\text{Re}^{1/8} P_r^{1/6} \left( P_r \frac{f}{f_0} - 1 \right)}}$$

where

$\frac{f_0}{f}$ = friction coefficient for a smooth tube at the same value of Reynolds' Number. This is known as Nunner's equation.

An analysis of hydraulically smooth pipes by Colburn gave

$$N_u = 0.023 \text{Re}^{0.8} P_r^{0.4}$$  \hspace{1cm} (A15.15)

For air in the atmospheric range, the Prandtl Number is near constant. Using the values at 20°C,

$$\mu = (17 + 0.045 \times 20) \times 10^{-6} = 17.9 \times 10^{-6} \text{ Ns/m}^2$$  \hspace{1cm} (Section 2.3.3)

$$k_a = 2.2348 \times 10^{-4} \left( 273.15 + 20 \right)^{0.8353} = 0.0257 \text{ W/(m°C)}$$  \hspace{1cm} (Table 15.1)

$$C_p = 1005 \text{ J/(kg°C)}$$

gives

$$P_r = \frac{C_p \mu}{k_a} = \frac{1005 \times 17.9 \times 10^{-6}}{0.0257} = 0.700$$  \hspace{1cm} (A15.16)

In order to illustrate the differences between the relationships quoted, Figure A15.2 has been produced for values of $P_r = 0.7$ and $f = 0.02$. There is reasonable agreement within the Von Kármán, Taylor and Rogers equations. The Nunner relationship produces Nusselt Numbers a little less than one half those given by the other authorities. As might be expected, the Colburn smooth tube equation gives much lower Nusselt Numbers and, hence, heat transfer coefficients.
Experimental evidence in mine airways (Mousset-Jones et al, 1987 and Danko et al, 1988) and also in scale models under controlled conditions (Deen, 1988) has indicated a better correlation with the Nunner equation than the others. Hence, this is the relationship suggested for use in underground openings at the present time. Ongoing research will no doubt produce further equations or algorithms for \( N_u \).

One further problem remains with equation (A15.14), the evaluation of \( f_0 \), the coefficient of friction for a hydraulically smooth tube at the same Reynolds Number. However, the approximation given in Section 2.3.6.3 gives acceptable results for the range of Reynolds' Numbers common in mine airways:

\[
f_0 = 0.046 \, \text{Re}^{-0.2}
\]

Substituting for \( f_0 \) in the Nunner equation and using \( Pr = 0.7 \) gives

\[
N_u = \frac{0.35 \, f \, \text{Re}}{1 + \frac{1.592}{\text{Re}^{0.125}} (15.217 \, f \, \text{Re}^{-0.2} - 1)}
\]

Figure A15.2 Relationships between Nusselt Number and Reynolds Number proposed by various authorities for \( Pr = 0.7 \) and \( f = 0.02 \).
APPENDIX A15.4

Derivation of the equation for latent heat of evaporation at a wet surface.

When any two gases, \(a\) and \(b\), are brought into contact then molecules of each gas will diffuse into the other at the interface. The fundamental law governing the rate of diffusion is

\[
m = D_v \frac{\partial \rho_a}{\partial x} \quad \text{kg/m}^2\text{s}
\]

where

\[
m = \text{rate of diffusion across unit area of gas } a \text{ into gas } b \quad (\text{kg/(m}^2\text{s)})
\]

\[
\rho_a = \text{density of gas } a \quad (\text{kg/m}^3)
\]

\[
x = \text{distance beyond the interface (m)} \text{ and}
\]

\[
D_v = \text{diffusion coefficient of gas } a \text{ into gas } b \quad (\text{m}^2/\text{s})
\]

(experimental results give this as \(25.5 \times 10^{-6} \text{ m}^2/\text{s}\) for the diffusion of water vapour into air (ASHRAE, 1985))

This is known as Fick’s Law and is analogous to Fourier’s law for heat conduction:

\[
q = -k \frac{\partial \theta}{\partial x} \quad \text{W/m}^2
\]

At a wet surface with unsaturated air flowing over it, water vapour is produced continuously by evaporation and will be transported by molecular diffusion through the laminar sublayer then, additionally, by eddy action through the transitional (buffer) and turbulent layers. The transfer of heat, momentum and mass are all affected in the same way by eddies at any given surface. Hence, the governing equations have the same form for all three. Just as we had

\[
\frac{d}{dt} \rho \frac{\partial q}{\partial x} = \text{W/m}^2 \quad \text{(see equation (15.6))}
\]

for heat flow across the boundary layers, so we can write an analogous equation for the migration of water molecules:

\[
m = h_m \left( \rho_{vws} - \rho_v \right) \quad \text{kg/(m}^2\text{s})
\]

where

\[
m = \text{mass of water vapour migrating from each square metre per second} \quad (\text{kg/(m}^2\text{s}) )
\]

\[
h_m = \text{mass transfer coefficient} \quad (\text{m/s})
\]

\[
\rho_{vws} = \text{density of water vapour at the wet surface} \quad (\text{kg/m}^3)
\]

\[
\rho_v = \text{density of water vapour in the mainstream} \quad (\text{kg/m}^3)
\]

The density difference \((\rho_{vws} - \rho_v)\) provides the potential or driving mechanism for the diffusion of mass.

Now from the general gas law

\[
e = \rho_v R_v T \quad \text{Pa} \quad \text{(see equation (3.11))}
\]

where

\[
e = \text{partial pressure of water vapour (Pa)}
\]

\[
R_v = \text{gas constant for water vapour (461.5 J/kg K)}
\]

\[
T = \text{absolute temperature (K)}
\]

Hence, equation (A15.20) becomes

\[
m = h_m \frac{1}{R_v T} \left( e_{vws} - e \right) \quad \frac{\text{kg}}{\text{m}^2\text{s}}
\]
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e is the vapour pressure within the main airstream and \( e_{ws} \) the vapour pressure at the wet surface. It is reasonable to assume that immediately adjacent to the liquid surface, the space will be saturated. \( e_{ws} \) is then also the saturated vapour pressure at the wet surface temperature, \( \theta_{ws} \).

Now, the general gas law for a mixture of air and water vapour gives

\[
\rho_m = \frac{P}{R_m T} \quad \text{Pa}
\]

where

- \( \rho_m \) = density of moist air (kg/m\(^3\))
- \( P \) = barometric pressure (Pa) and
- \( R_m \) = gas constant for moist air (J/(kg K))

However, for the atmospheric range in mines, \( R_m \) differs only slightly from the gas constant for dry air, \( R_a = 287.04 \text{ J/(kg K)} \) [Section 14.2.4]. Hence, we can express equation (A15.21) as

\[
m = h_m \frac{\rho_m}{R_v T} \frac{1}{\rho_m} (e_{ws} - e)
\]

\[
= h_m \frac{\rho_m}{R_v T} \frac{R_a T}{P} (e_{ws} - e)
\]

\[
= h_m \rho_m 0.622 \frac{(e_{ws} - e)}{P} \quad \text{kg/m}^2\text{s}
\]

(A15.22)

as

\[
\frac{R_a}{R_v} = \frac{287.04}{461.5} = 0.622
\]

Now let us turn our attention to the mass transfer coefficient, \( h_m \). In 1934, Chilton and Colburn, investigating the analogy between heat transfer and mass diffusion, produced a relationship between the convective heat transfer coefficient, \( h_c \), and the mass transfer coefficient, \( h_m \). In its most compact form (ASHRAE, 1985), this can be expressed as

\[
h_m = \frac{h_c}{C_p \rho_m} \left[ \frac{P_r}{S_c} \right]^{2/3} \quad \text{m/s}
\]

(A15.23)

where

- \( C_p \) = specific heat of air (J/kg K)
- \( P_r = \frac{\mu_a C_p}{k_a} \) is the Prandtl Number for air (Appendix A15.3)
- \( \mu_a \) = dynamic viscosity of air (Ns/m\(^2\))
- \( k_a \) = thermal conductivity of air (W/(m°C))

and \( S_c \) is yet another dimensionless number (Schmidt Number) = \( \frac{\mu_a}{\rho_a D_v} \)

Both the Prandtl and Schmidt dimensionless numbers remain reasonably constant over the range of air pressures and temperatures found in subsurface facilities.

Using the values for air at 20°C,

\[
\mu_a = 17.9 \times 10^{-6} \quad \text{Ns/m}^2; \quad k_a = 0.0257 \quad \text{W/(m°C)}
\]

\[
C_p = 1010 \quad \text{J/(kgK)} \quad \text{(mid-range value for moist air)};
\]

\[
\rho_a = 1.2 \text{ kg/m}^3 \quad \text{(standard air density)} \quad \text{and}
\]

\[
D_v = 25.5 \times 10^{-6} \quad \text{m}^2/\text{s} \quad \text{for the diffusion of water vapour into air,}
\]
Heat flow into subsurface openings

Combining equations (A15.22 and A15.23) gives

\[ m = \frac{h_c}{C_p \rho_m} \left[ \frac{P_r}{S_c} \right]^{2/3} \rho_m 0.622 \left( \frac{e_{ws} - e}{P} \right) \]

and, inserting the numerical values for \( P_r \) and \( S_c \),

\[ m = h_c 1.130 \times \frac{0.622}{C_p} \left( \frac{e_{ws} - e}{P} \right) \quad \text{kg m}^{-2} \text{s}^{-1} \quad (A15.24) \]

Now the latent heat emitted in the vapour when \( m \) kg of surface water is evaporated is

\[ q_L = m L_{ws} \quad \text{W m}^{-2} \quad (A15.25) \]

where

\[ L_{ws} = \text{latent heat of evaporation at the temperature of the wet surface (see equation (14.6))} \]

\[ = (2502.5 - 2.386 \theta_{ws})10^3 \quad \text{J/kg} \]

Substituting for \( m \) from equation (A15.24) gives

\[ q_L = h_c 1.130 \times \frac{0.622 L_{ws}}{C_p} \left( \frac{e_{ws} - e}{P} \right) \quad \text{W m}^{-2} \quad (A15.26) \]

or, using the value \( C_p = 1010 \text{ J/kgK} \) for moist air,

\[ q_L = 0.0007 h_c L_{ws} \left( \frac{e_{ws} - e}{P} \right) \quad \text{W m}^{-2} \quad (A15.27) \]

Equation (A15.27) may also be written as

\[ q_L = h_e \left( \frac{e_{ws} - e}{P} \right) \quad \text{W m}^{-2} \quad (A15.28) \]

where the evaporative heat transfer coefficient, \( h_e \), is given as

\[ h_e = 0.0007 h_c \frac{L_{ws}}{P} \quad \text{W m}^{-2} \text{Pa} \quad (A15.29) \]

The ratio \( h_e/h_c \) is known as the Lewis Ratio (LR).

Using the latent heat of evaporation of water at 20°C (2455 x 10^3 J/kg) and a barometric pressure of 100 x 10^3 Pa gives

\[ \text{LR} = \frac{h_e}{h_c} = 0.017 \quad \text{C} \quad (A15.30) \]

The Lewis ratio changes slowly through the atmospheric range of pressures and temperatures.

A correlation with psychrometric relationships occurs in equation (A15.26). The group

\[ 0.622 L_{ws}/C_p \]

is the reciprocal of the psychrometric “constant” introduced in Section (14.4.2).